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! Bond portfolio/cash matching problem in LINGO. Given:
  cash needs in each of a series of future periods, what
  collection of high quality bonds should we buy now,
  so to meet these needs with certainty (the bonds do not default)?;
! We want to minimize initial cash needed;
! Keywords: @SEMIC, Bond portfolio, Cash flow matching, Covering, Defeasance,
    LINGO, Maturity, Minimum buy, Semi-continuous ;
SETS:
   BOND :
      MATAT,    ! Maturity period;
      PRICE,    ! Price of bond;
      CAMNT,    ! Coupon payment each period;
      BUY,      ! Amount to buy;
      BUYMIN,   ! Minimum that must be bought if any;
      BUYMAX;   ! Maximum would ever buy;
   PERIOD:
      NEED,     ! Cash needed each period;
      SINVEST; ! Short term investment each period;
ENDSETS

DATA:
   STRTE = 0.01;          ! Short term interest rate;
    BOND =   A      B       C;
   MATAT =   6,    13      15;   ! Years of maturity;
   PRICE = 0.980, 0.965, 0.99;  ! Bond purchase prices in 1000's;
   CAMNT = 0.060, 0.065, 0.064; ! Bond coupon payments in 1000's;
   BUYMIN=   5      5      9;
   BUYMAX=  140     140    130;
   NEED = 10, 11, 12, 14, 15, 17, 19, 20, 22, 24,
          26, 29, 31, 33, 15; ! Cash needs in 1000's for each period;
ENDDATA

SUBMODEL CHUZBNDS:
! Minimize the total investment required to generate
  the stream of future cash needs;
   MIN = LUMP;
! First period is special, must cover first need,
   plus buy bonds;
   LUMP = NEED( 1) + SINVEST( 1) +
    @SUM( BOND( b): PRICE( b) * BUY( b));
! For subsequent periods;
   @FOR( PERIOD( I)| I #GT# 1:
 ! Sources of cash = uses of cash, or
   interest payments + maturities = needs to be covered;
    @SUM( BOND( b)| MATAT( b) #GE# I:
     CAMNT( b) * BUY( b)) +
      @SUM( BOND( b)| MATAT( b) #EQ# I:
     BUY( b)) +
     ( 1 + STRTE) * SINVEST( I - 1) =
      NEED( I) + SINVEST( I);
   );
! Can only buy integer bonds;
   @FOR( BOND( b): @GIN( BUY( b)));
! Enforce Min Buy quantities with semi-continuous constraint.
 If we buy any, we must buy >= BUYMIN;
   @FOR( BOND( b):  @SEMIC( BUYMIN( b), BUY( b), BUYMAX( b)));

ENDSUBMODEL

CALC:
! @GEN( CHUZBNDS);  ! Generate display of model in scalar form;
 @SOLVE( CHUZBNDS); ! Solve the model;

 ! Write a little report;
 @WRITE(' Initial Cash Needs in Period 1= ', @FORMAT( LUMP, '9.2f'), @NEWLINE( 2));
 @WRITE(' Bond  Maturity  Price    Coupon  Units_to_buy', @NEWLINE(1));
  @FOR( BOND( b):
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    @WRITE('  ', Bond(b),'     ',  @FORMAT( MATAT( b),'3.0F')
                        ,'      ', @FORMAT( PRICE( b),'6.4F')
                        ,'   ',    @FORMAT( CAMNT( b),'6.4F')
                        ,'     ',  @FORMAT(   BUY( b),'3.0F')
                        , @NEWLINE(1));
      );

ENDCALC
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