

How to Add Optimization to Planning Under Uncertainty

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Based on a presentation at INFORMS Conference Philadelphia, PA

31 October 2015

Keywords: Uncertainty, Stochastic Optimization, Tornado diagrams, Fuzzy optimization, Parametric analysis, Scenario planning.





We have a Planning Model with Optimization:

Maximize some objective, e.g., selling_price*volume – production_cost/unit * volume;

subject to various constraints, e.g., production at each source \leq capacity at that source, supply to each region \geq demand at that region;

We are not sure of the values of various coefficients in the model, e.g., selling price per unit, cost of raw materials, demand, etc.

What should/can we do? What convenient tools are available, especially in LINGO and What's*Best*! ?



Approaches: Simple to Fancy

Methods that require no additional information beyond the original model:

Range analysis and dual prices: Works only for Linear Program Models.

Parametric analysis:Try a range of values for each uncertain parameter.K-Best solutions:Solve for the K best solutions. Which seems most realistic?

Methods that require only Scenario value information for uncertain parameters

<u>Tornado diagrams</u> :	Which parameter uncertainties have biggest effect?
Fuzzy optimization	Analyze all possible outcomes.
Robust optimization	Worry about worst possible outcome.
Data tables in Excel	Automatically generate all outcomes for 1 or 2 parameters.
Scenario feature of Excel	Enumerate possible scenarios for up to 32 parameters.

Methods that require a distribution of uncertain parameters

Chance Constrained Programs:Robust optimization with probabilities.Value at Risk, Conditional Value at RiskHow bad is the 5% risk case?Stochastic OptimizationWhat is best way to hedge/prepare all possibilities?

Measuring the cost of uncertainty

<u>Value of More Accurate Forecasts</u>, <u>Value of Modeling Uncertainty</u>,

Can't eliminate variability, but we can know it.

Given the available forecast quality.



```
a) Deterministic Case:
    ! The objective is to maximize profit;
    MAX = 20*ASTRO + 30*COSMO;
        ASTRO <= 60; ! Astro line capacity;
        COSMO <= 50; ! Cosmo line capacity;
    ! Labor usage <= labor available;
        1*ASTRO + 2*COSMO <= 120;</pre>
```

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b) Parametric/Uncertain/Scenario Case:
MAX = PAM(1) *ASTRO + PAM(2) *COSMO;
ASTRO <= PAM(3); ! Astro demand;
COSMO <= PAM(4); ! Cosmo demand;
PAM(6) *ASTRO + PAM(7) *COSMO <= PAM(5);</pre>
```

1) APRFT 2) CPRFT 3) ALCAP 4) CLCAP 5) LABORAVAIL 6) ALBRUSE 7) CLBRUSE; LINDO SHSTEMS INC.

Dual Prices and Range Analysis of an LP

Global optim	al solution found.		
Objective v	alue: 2100	0.000	
Variable	Value	Reduced Cost	
ASTRO	60.00000	0.00000	
COSMO	30.00000	0.00000	
Row	Slack or Surplus	Dual Price	
1	2100.000	1.000000	
2	0.00000	5.00000	! More Astro Line capacity is worth \$5/unit;
3	20.00000	0.00000	! More Cosmo Line capacity is worth \$0/unit
4	0.00000	15.00000	! More Labor capacity is worth \$15/unit

! Click on: Solver -> Range

Ranges in which the basis is unchanged:

	Objective Co	efficient Ranges:	
	Current	Allowable	Allowable
Variable	Coefficient	Increase	Decrease
ASTRO	20.00000	INFINITY	5.000000
COSMO	30.00000	10.00000	30.00000
	Righthan	d Side Ranges:	
	Current	Allowable	Allowable
Row	RHS	Increase	Decrease
2	60.00000	60.00000	40.00000
3	50.00000	INFINITY	20.00000
4	120.0000	40.00000	60.00000





KBest Solutions

SETS: ITEM: WGT, VAL, Y; !Each item has a wgt, value, yes/no var; ENDSETS DATA:

ITEM	WGT	VAL =
ANT_REPEL	1	2
SIX_PACK	3	9
BLANKET	4	3
BRATWURST	3	8
BROWNIE	3	10
FRISBEE	1	6
SALAD	5	5
WATERMELON	10	20;
CAP = 15;		

ENDDATA





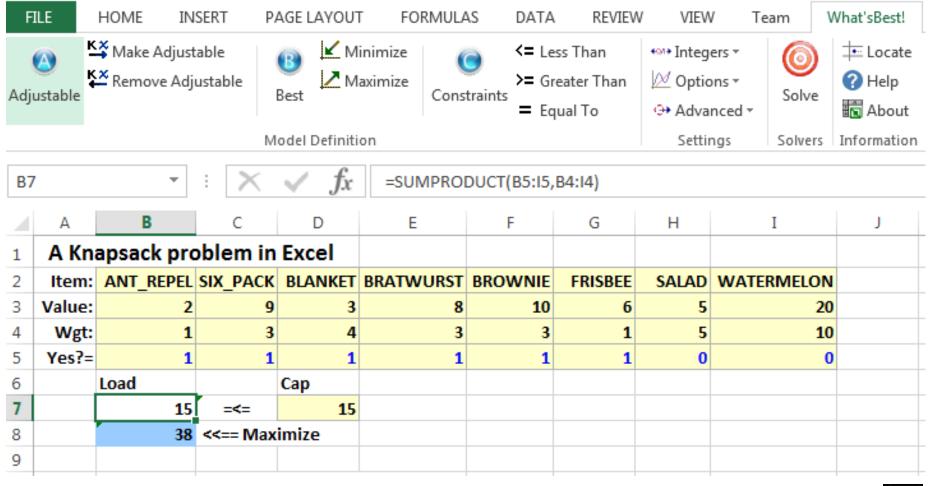
K Best Solutions in LINGO

🔁 Lir File

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3	Value:					8 10	6	5		20		
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23 24							8	Spec	ify Reporting Cell	s	LINDO SYSTEMS	









K Best Solutions in What's Best!

A B	C	D	E	F	G	H	I	J	
	~	<u> </u>	-		-		•		

K-Best Solution Displayed in Spreadsheet: 1

REPORTING CELLS

9	Run										
10			ObjVal	ANT_REPEL	SIX_PACK	BLANKET	BRATWURST	BROWNIE	FRISBEE	SALAD	WATERMELON
11											
12	-	1-	38	1	1	1	1	1	1	0	0
13	-	2-	38	0	1	0	1	1	1	1	0
14	-	3-	38	1	0	0	0	1	1	0	1
15	-	4-	37	1	1	0	0	0	1	0	1
16	-	5-	36	1	0	0	1	0	1	0	1
17	-	6-	36	0	0	0	0	1	1	0	1
18	-	7-	36	0	1	1	1	1	1	0	0
19	-	8-	35	1	1	0	1	1	1	0	0
20											

End of Report



Parametric Analysis: Markowitz Portfolio

	t Frontier Portfolio Calculation	(See	PortEfFront9a.lng)
The p	oossible investments:		
CD=	risk-free rate,		
VG040=	SP500 stock index,		
VG058=	Insured long term tax exempt,		
VG072=	Pacific stock index		
VG079=	European Stock index,		
VG102=	Tax managed cap appreciation,		
VG533=	Emerging markets.		

After tax

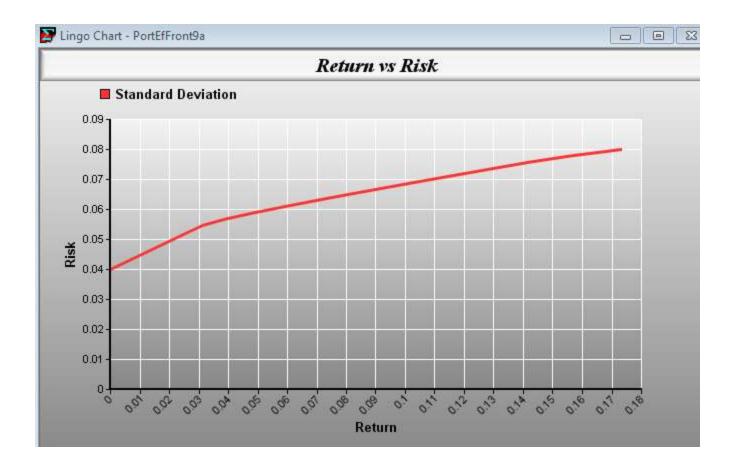
Target	Risk(1 s	d) Por	rtfolio	compos	ition			
Return	1-Yr	CD	VG040	VG102	VG058	VG079	VG072	VG533
0.04000	0.0000	1.0000						
0.04500	0.0106	0.6483	0.0085	0.0265	0.2496	0.0413	0.0257	
0.05000	0.0212	0.2967	0.0170	0.0530	0.4993	0.0827	0.0513	
0.05500	0.0321		0.1052		0.6645	0.1316	0.0987	
0.06000	0.0541		0.0282		0.5349	0.0926	0.1999	0.1443
0.06500	0.0806				0.4046		0.2863	0.3091
0.07000	0.1087				0.2155		0.3535	0.4310
0.07500	0.1376				0.0264		0.4207	0.5528
0.08000	0.1733							1.0000

Input Data Used:

Expected ret/yr: 0.0400 0.0600 0.0600 0.0500 0.0650 0.0700 0.0800 Stdev in ret/yr: 0.0000 0.0811 0.0911 0.0370 0.1010 0.1252 0.1733



Parametric Analysis



! Graph it as is done by Finance folks; @CHARTCURVE('Return vs Risk', 'Return', 'Risk', 'Standard Deviation', VOUT, VINP);



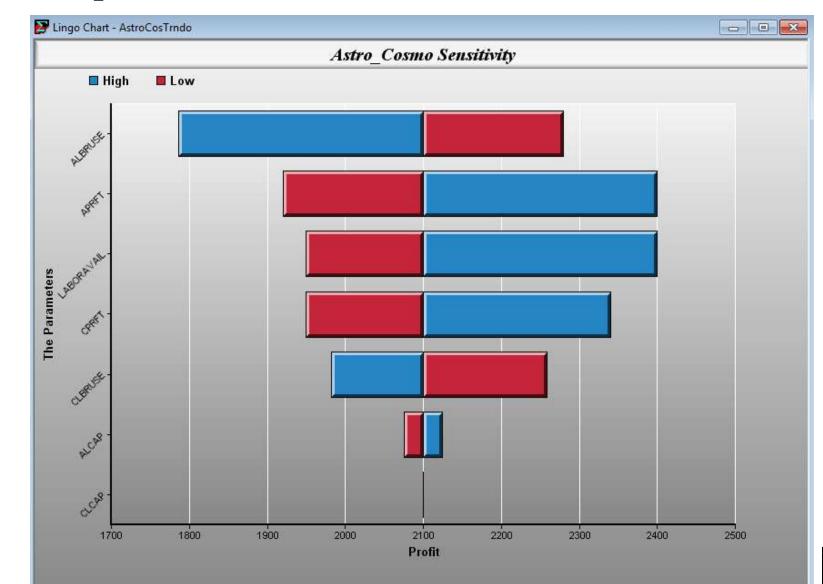
Tornado Diagram Analysis

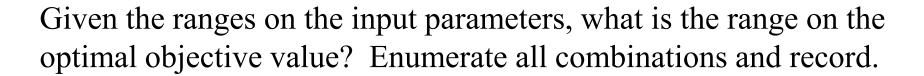
Recall: Parametric/Uncertain/Scenario Case: MAX = PAM(1) * ASTRO + PAM(2) * COSMO;ASTRO <= PAM(3); ! Astro demand; $COSMO \le PAM(4)$; ! Cosmo demand; $PAM(6) * ASTRO + PAM(7) * COSMO \le PAM(5);$ DATA: ! Names of the parameters; PSET =2 3 4 5 6 1 7 ; APRFT CPRFT ALCAP CLCAP LABORAVAIL ALBRUSE CLBRUSE; ! The median or base case values for the parameters; PMED =20 30 60 50 1 120; 2 ! Plausible low values for the parameters; PLO =25 55 45 0.8 1.7 110; 17 ! Plausible high values for the parameters; PHI =25 38 65 2.3 60 1.4 140; ! For this parameter set we will see that LABORAVAIL has the greatest effect on bottom line uncertainty. CLCAP has the least effect (none) on bottom line uncertainty; **ENDDATA**





@CHARTTORNADO('Astro_Cosmo Sensitivity', 'Profit', 'The Parameters', BASE, 'High', RESULTHI, 'Low', RESULTLO);





Sensitivity Analysis for an Optimization Problem with Various Lo and Hi/Fuzzy Parameter Values:

Fuzzy Optimization

			_,					
Iter	APRFT	CPRFT	ALCAP	CLCAP	LABRAVL	ALBRUSE	CLBRUSE	Profit
1	17.00	25.00	55.00	45.00	110.00	0.80	1.70	1905.59
2	17.00	25.00	55.00	45.00	110.00	0.80	2.30	1652.39
3	17.00	25.00	55.00	45.00	110.00	1.40	1.70	1531.79
4	17.00	25.00	55.00	45.00	110.00	1.40	2.30	1293.70
5	17.00	25.00	55.00	45.00	140.00	0.80	1.70	2060.00
• • •								
124	25.00	38.00	65.00	60.00	110.00	1.40	2.30	1938.91
125	25.00	38.00	65.00	60.00	140.00	0.80	1.70	3592.06
126	25.00	38.00	65.00	60.00	140.00	0.80	2.30	3078.91
127	25.00	38.00	65.00	60.00	140.00	1.40	1.70	2958.57
128	25.00	38.00	65.00	60.00	140.00	1.40	2.30	2434.57
The	optima	l value	falls	in the	range:			
[Pro	fitto	ProfitH	i = r	1293	6957	3592	05881	

ProfitLo, ProfitHi] = [1293.6957, 3592.0588] Iteration= 4 125 LINDO SYSTEMS ING

Data Tables in Excel

DAV	MENT	• : X	$\sqrt{f_x} = -$	PMT(E5,F5,D5)				
			0			_		
	A	В	С	D	E	F	G	Н
1	Illustra	ation of a T	wo-Dimer	nsional D	ata Table	in Excel		
2	to com	npute the mor	nthly mortgag	e payment	for various i	nterest rate and	d term scena	arios.
3		(Col input)	(Row input)	Principal	Monthly	Number of	Monthly	Total
4		Yearly rate	<u>Years</u>	<u>Amount</u>	<u>rate</u>	<u>months</u>	Payment	payments
5		0.06	15	200000	0.005	180	1687.71	25315.70
6								
7								
8			Scenario tabl	e of Month	ly Payment a	is a function of	Yearly rate a	and Years
9		Output cell		Years				
10		\$1,687.71	1	5	10	30		
11	Yearly	0.02		3505.55	1840.269			
12	interest		16938.74	3593.74	1931.215			
13	rate	0.04	17029.981	3683.3	2024.903	954.830591		
14		0.05	17121.496	3774.25	2121.31	1073.64325		
15		0.06	17213.286	3866.56	2220.41	1199.10105		
16		0.07	17305.349	3960.24	2322.17	1330.60499		
17		0.08	17397.686	4055.28	2426.552	1467.52915		
18		0.09	17490.295	4151.67	2533.515	1609.24523		
19		0.1	17583.177	4249.41	2643.015	1755.14314		
20								
21	Steps:							
22	-	ruct the core/s	-		-	e, just row 5.		
23	2) Enter t	the scenario d	lata, B11:B19,	, and C10:F	10			
24	3) Do a C	opy and Paste	e-Link of G5 to	o B10 to ide	ntify the out	put cell.		
25	4) Highlig	ght the table B	10:F19					
26	5) Click o	n DATA -> W	hat-if-analysis	s -> Data Ta	ble, and			
27	6) Enter (C5 as the Row	Input cell, an	d				
28	B5 as	the Column I	nput cell.					
29								

F5		• : X	$\checkmark f_x$	=YEARS*12						
	А	В	С	D	E	F	G	Н	Ι	J
1	Illustrat	tion of Sce	nario M	anager ir	n Excel					
2	to comp	oute the mont	thly mortga	age paymer	nt for various	s interest rate a	nd term scena	rios.		
3				Principal	Monthly	Number of	Monthly	Total payments		
4		Yearly rate	<u>Years</u>	<u>amount</u>	<u>rate</u>	<u>months</u>	Payment	payments		
5		0.05	25	200000	0.004167	300	1169.18	29229.50		
6							Scenario Manage	r	2	×
7							S <u>c</u> enarios:			
8	Steps:						CASE1506 CASE2006	*	<u>A</u> dd	
9	1) Constru	ict the core/si	ingle scena	rio model.	In this examp	ole, just row 5.	CASE3006 CASE2505		Delete	
10							Edit			
11										
12									Merge	
13								-	S <u>u</u> mmary	
14							Changing coller	ener.ecer		
15							Changing cells: Comment:	\$B\$5:\$C\$5 Created by on 10/24/20)15	
16								Modified by on 10/24/2 Modified by on 10/26/2	2015	
17								,		





Setting:

- 0) We make a decision, e.g., inventory levels, investments, etc.
- 1) Nature makes a random decision.

Robust Optimization

First identify a set of possible scenarios/outcomes for the random variables. In (0),

Choose the decision the maximizes the profit, subject to being feasible for every possible scenario.

```
Slightly more mathematically:

minimize f_0(x)

subject to

For every constraint i:

For every scenario s:

f_i(x, u_s) \ge 0;
```

(or maximize, as desired)





Example : Portfolio investment a) Deterministic Case:

! Maximize end-of-period wealth; MAX = 1.089*ATT + 1.214*GMC + 1.235*USX + 1.05*XTBILL; ! We have \$1M to start;

ATT + GMC + USX + XTBILL = 1;

b) Parametric/Uncertain/Scenario Case:

MAX=PAM(1)*ATT + PAM(2)*GMC + PAM(3)*USX + 1.05*XTBILL; ATT + GMC + USX + XTBILL = 1; PAM(1)*ATT + PAM(2)*GMC + PAM(3)*USX + 1.05*XTBILL >= TARGET;



Portfolio Example, Scenarios

! Some equally likely scenarios of future values of each of the instruments per \$1 invested today

ATT	GMC	USX ;
1.300	1.225	1.149
1.103	1.290	1.26
1.216	1.216	1.419
0.954	0.728	0.922
0.929	1.144	1.169
1.056	1.107	0.965
1.038	1.321	1.133
1.089	1.305	1.732
1.090	1.195	1.021
1.083	1.390	1.131
1.035	0.928	1.006
1.176	1.715	1.908;





Robust Optimization

<pre>DATA: TBILLGF = 1.05; ! Risk free growth factor, e.g., for TARGET = 1.01; ! Target growth factor; SCENE = 112; ! Number of scenarios; ! Our investment opportunities, in addition to T Bill ASSET= ATT GMC USX; </pre>	
ENDDATA	
<pre>NS = @SIZE(SCENE); ! Number scenarios; ! Stage 0: Choose the X's and AVG;</pre>	
<pre>! Budget constraint at beginning; [BUD] @SUM(ASSET(J): X(J)) + XTBILL = 1; @FOR(SCENE(s): ! Compute R(s) = value of total portfolio under scena X(i) = amount invested in instrument i; R(s) = @SUM(ASSET(j): GF(s, j) * X(j));</pre>	
<pre>! Compute expected value of ending position, assuming all scenarios equally likely; AVG = @SUM(SCENE(s): R(s)) / NS;</pre>	
<pre>! Robustness constraints: We want to beat the target in every scenari @FOR(SCENE(s): R(s) >= TARGET;);</pre>	-0;
! A reasonable objective: Maximize average return; MAX = AVG;	LINDO SYSTEMS INC.



Robust Optimization

Variable	Value
AVG	1.078841
TBILLGF	1.050000
TARGET	1.030000
XTBILL	0.8437500
X(ATT)	0.00000
X(GMC)	0.00000
X(USX)	0.1562500
R(1)	1.065469
R(2)	1.082813
R(3)	1.107656
R(4)	1.030000
R(5)	1.068594
R(6)	1.036719
R(7)	1.062969
R(8)	1.156563
R(9)	1.045469
R(10)	1.062656
R(11)	1.043125
R(12)	1.184063

ATT	GMC	USX ;
1.300	1.225	1.149
1.103	1.290	1.26
1.216	1.216	1.419
0.954	0.728	0.922
0.929	1.144	1.169
1.056	1.107	0.965
1.038	1.321	1.133
1.089	1.305	1.732
1.090	1.195	1.021
1.083	1.390	1.131
1.035	0.928	1.006
1.176	1.715	1.908;

Recall:





Chance Constrained Programing: we are allowed to violate certain specified constraints with a specified (typically low) probability;

```
! Chance constraints;
  @FOR( SCENE( s):
! ZSAT( s) = 1 if we satisfy constraint in scenario s;
  @BIN( ZSAT(s)); ! It is 0 or 1;
  R( s) >= ZSAT( s) * TARGET;
  );
```

- ! We want to beat the target this fraction of the time ; @SUM(SCENE(s): ZSAT(s)) / NS >= PROBCC;
- ! A reasonable objective: Maximize average return; MAX = AVG; LINDO 5457EM5 INC



Chance Constrained Optimization

Variable AVG TBILLGF TARGET PROBCC XTBILL X(ATT) X(GMC)	Value 1.225009 1.050000 1.030000 0.8300000 0.000000 0.000000 0.4577465
X(USX)	0.5422535
R(1)	1.183789
R(2)	1.273732 1.326077
R(3) R(4)	0.8331972
R(5)	1.157556
R(6)	1.030000
R(7)	1.219056
R(8)	1.536542
R(9)	1.100648
R(10)	1.249556
R(11)	0.9702958
R(12)	1.819655
ZSAT(1)	1.000000
ZSAT(2)	1.000000
ZSAT(3)	1.000000
ZSAT(4)	0.00000
ZSAT(5)	1.000000
ZSAT(6)	1.000000
ZSAT(7)	1.000000
ZSAT(8)	1.000000 1.000000
ZSAT(9) ZSAT(10)	1.000000
ZSAT(10) ZSAT(11)	0.000000
ZSAT(11) ZSAT(12)	1.000000
	_

Recall:

ATT	GMC	USX ;
1.300	1.225	1.149
1.103	1.290	1.26
1.216	1.216	1.419
0.954	0.728	0.922
0.929	1.144	1.169
1.056	1.107	0.965
1.038	1.321	1.133
1.089	1.305	1.732
1.090	1.195	1.021
1.083	1.390	1.131
1.035	0.928	1.006
1.176	1.715	1.908;



What Should Our Objective Criterion be Under Uncertainty?

Desirable Features of a Utility Function:

- 1) More is better: An additional dollar is always appreciated, no matter how much we have already.
- 2) Concavity: Twice as much is not twice better. The (n+1)st dellar is no more valuable then the nth

The $(n+1)^{st}$ dollar is no more valuable than the n^{th} dollar.





To use VaR, you must specify two numbers:

1) a probability threshold, typically 5% (or 1%), beyond which you care about bad outcomes.

2) an interval of time, typically one day or ten days, over which you are concerned about losing money,

VaR = amount of loss in one day that has at most a 5% (or 1%) probability of being exceeded.

VaR is a method recommended as part of the Basel Accord for measuring the risk of the portfolios of European banks. Banks must hold capital reserves proportional to their risk, e.g., as measured by VaR.

Solution:	Variable TBILLGF RHO NS	Value 1.050000 0.1670000 12.00000	! Some equally likely scenarios of the future values of each of the instruments per \$1 invested today;
	BIGM	1.180000	ASSET= ATT GMC USX ;
	XTBILL	0.00000	GF = ! Growth Factors, each investment
	X(ATT)	0.00000	1.300 1.225 1.149
	X(GMC)	1.000000	1.103 1.290 1.26
	X(USX)	0.00000	1.216 1.216 1.419
	AVG	1.213667	
	Т	1.107000	0.954 0.728 0.922
	R(1)	1.225000	0.929 1.144 1.169
	R(2)	1.290000	1.056 1.107 0.965
	R(3)	1.216000	1.038 1.321 1.133
	R(4)	0.7280000	1.089 1.305 1.732
	R(5)	1.144000	1.090 1.195 1.021
	R(6)	1.107000	1.083 1.390 1.131
	R(7)	1.321000	
	R(8)	1.305000	1.035 0.928 1.006
	R(9)	1.195000	1.176 1.715 1.908;
	R(10)	1.390000	
	R(11) R(12)	0.9280000 1.715000	LINDO SYSTEMS INC. 💻

Conditional Value at Risk

CVaR requires us to specify a risk tolerance ρ , e.g., 5%. If the random variable *w* is the final wealth of the portfolio, then CVaR chooses a portfolio and VaR threshold, *t*, so as to maximize a weighted combination of: the final portfolio value, the VaR value, and minus the expected amount by which the final portfolio falls short of the VaR target. Optionally, we may specify an expected return preference $\alpha \ge 0$. Algebraically, the CVaR objective is: Max $\alpha E(w) + \rho t - E(\max[0, t - w])$.



Conditional Value at Risk, Details

```
! Compute portfolio value, R(s), under each scenario s;
@FOR(SCENE(S): R(S) = @SUM(ASSET(J):VE(S,J) * X(J));
! Measure deviations from target T;
DVL(S) - DVU(S) = T - R(S);
);
! Compute expected value of ending position;
[DEFAVG] AVG = @SUM(SCENE(s): PRB(s) * R(s));
! Ending value >= target;
[RET] AVG >= TARGET;
! Minimize conditional value at risk;
[OBJV] MAX = OBJ; OBJ = ALPHA*AVG + RHO*T - @SUM(SCENE(s): PRB(s)* DVL(s));
! Notice that as long as the fraction of the scenarios with
```

R(s) < T is < RHO, we (and the optimizer) can increase T;



Portfolio's: Various Objectives

```
! Scenario portfolio model with various possible objectives.
See end of model
LINGO will automatically choose the appropriate solver:
 Linear, Quadratic/Second Order Cone, or Nonlinear;
SETS:
SCENE: PROB, R, DVU, DVL, DV2, DV3, DV1;
ASSET: X; ! X(j) = amount to invest in asset j;
SXA( SCENE, ASSET): GF ;
ENDSETS
DATA:
TBILLGF = 1.05; ! Risk free growth factor, e.g., for money invested in Treasury Bills;
TARGET = 1.15; ! Target growth factor;
SCENE = 1..12; ! Number of scenarios;
! Our investment opportunities, in addition to T Bills;
ASSET=
          ATT
                  GMC
                          USX ;
! Some equally likely scenarios of the future
 values of each of the intruments per $1 invested today;
        ! The yearly Growth Factors for each investment;
GF =
        1.300
                1.225
                        1.149
        1.103
              1.290
                      1.260
        1.216 1.216 1.419
       0.954 0.728
                      0.922
        0.929
                      1.169
               1.144
        1.056
               1.107 0.965
        1.038 1.321
                      1.133
        1.089
               1.305
                       1.732
                       1.021
        1.090
               1.195
        1.083 1.390
                       1.131
        1.035 0.928
                      1.006
                1.715
        1.176
                        1.908;
! All scenarios equally likely;
PROB = .083333 .083333 .083333 .083333 .083333 .083333
                                                                 LINDO SYSTEMS INC.
      .083333 .083333 .083333 .083333 .083333 .083333;
```



```
@FREE( AVG);
! Stage 0: Choose the X's and AVG;
! Budget constraint;
[BUD] (SUM(ASSET(J): X(J)) + XTBILL = 1;
! Target ending value;
[RET] AVG >= TARGET;
! Stage 1:
@FOR( SCENE( S):
     @FREE( R( S));
! Compute R(s) = value of total portfolio under scenario s.
   X(i) = amount invested in instrument i;
   R(s) = O(SUM)(ASSET(j)): GF(s, j) * X(j)) + XTBILL*TBILLGF;
! Measure deviations up and below from average;
   DVU(s) - DVL(s) = R(s) - AVG;
        );
! Compute expected value of ending position;
AVG = (SUM(SCENE(S)) : PROB(S) * R(S));
```



Portfolios, Various Objectives, I

! Set objective to one of the following...;

```
! Linear objectives;
    1) Minimum absolute deviation(MAD) in return;
!
I
   MIN = @SUM(SCENE(s): PROB(s) * (DV1(s)));
!
   2) Downside risk;
I
  MIN = @SUM(SCENE(s): PROB(s) * DVL(s));
! Quadratic objectives;
   3) Simple variance;
ļ
   MIN = (SUM(SCENE(s): PROB(s) * (DV1(s))^2);
ļ
   4) Semi-variance, or squared downside risk;
!
!
   MIN = @SUM(SCENE(s): PROB(s) * DVL(s)^2);
! Conic objective,
    5) Value-at-Risk, assuming Normal Distribution
 and a 5% risk tolerance;
   SD^2 \ge (SUM(SCENE(s):(DV1(s))^2);
ļ
! Maximize a weighted combination of mean less SD;
1
   MAX = AVG - 1.645 * SD;
```



```
Portfolios, Various Objectives, II
! Nonlinear objective;
   6) Absolute deviations raised to 3rd power (We hate large deviations);
    MIN = @SUM(SCENE(s): PROB(s) * (DV1(s))^3);
I
   7) Tell Global solver to trust us that we know the objective is convex;
!
   Deviations raised 3rd power ( We hate large deviations);
I
   CUROBJ >c= (OSUM(SCENE(s): PROB(s) * (DV1(s))^3);
1
   MIN = CUROBJ;
   8) To illustrate the generality of Conic/SOC capability,
1
  3rd power objective converted to SOC form;
   @FOR( SCENE(s):
DV2(s) >= DV1(s)^{2};
  Force DV3 to = the third power;
      DV3(s) * DV1(s) >= DV2(s)^2;
       );
  Sum of the 3rd powers;
   MIN = @SUM(SCENE(s): PROB(s)*DV3(s));
  In fact, any power > 1 can be converted to SOC;
```



0

Portfolios, Various Objectives,

Case 4 (Semi-variance): Global optimal solution found. Objective value: Elapsed runtime seconds: Model is convex quadratic

0.7155100E-02 0.08

Variable	Value
X(ATT)	0.1903418E-06
X(GMC)	0.1068679
X(USX)	0.4470278





Portfolios, Various Objectives

Case 6 (NLP, Deviations to 3rd power) Local optimal solution found. Objective value: Elapsed runtime seconds:

0.4715426E-02 0.09

Variable	Value
X(ATT)	0.3371926
X(GMC)	0.3272703
X(USX)	0.1802045





Portfolios, Various Objectives

Case 8 (Deviations to 3rd power as SOC): Global optimal solution found. Objective value: Elapsed runtime seconds: Model is a second-order cone

0.4715455E-02 0.11

Variable	Value
XTBILL	0.1553326
X(ATT)	0.3371927
X(GMC)	0.3272697
X(USX)	0.1802050





The "gold standard" for planning under uncertainty.



How is SP Information Stored in the SpreadSheet?

All information about the SP features is stored explicitly/openly on the spreadsheet.

- Core model is a regular deterministic What's*Best*! or LINGO model. You can plug in regular numbers in a random cell to check results.
- 2) Staging information is stored in Decisions: WBSP_VAR(stage, cell_list) and Random variables: WBSP_RAND(stage, cell_list);
- 3) Distribution specification is stored in WBSP_DIST_distn(table, cell_list); where distn specifies the distribution, e.g., NORMAL cell.
- 4) Sample size for each stage is stored in WBSP_STSC(table);
- 5) Cells to be reported are listed in WBSP_REP(cell_list) or WBSP_HIST(bins, cell); LINDO SYSTEMS INC.



The "Core Model" is a completely valid Excel model.

If you are doing neither simple optimization nor SP, you can do complete "What-If" analysis with it as a valid deterministic model.

If you have not turned on SP, you can do simple optimization with it like any deterministic What'sBest model.



	Stochastic	Optir	nizati	on: I	New	vsve	endor in	Wha	aťs.	Bes	st!
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1	Stochastic Optimization of Newsve	endor, N	ormal Dei	mand <i>(L</i>	inear	versior	ı)				
2	Given all costs and prices, in	-		· ·			Stochastic Support				
3	Stage 0 we must decide how many newspaper			lles in							
4 5	Stage 1, in the beginning, unknown demand is Stage 1, at the end, we compute our sales and			liy in			Use Stochastic Modeli	ng	🗌 Use	Simulation F	Format
6	1) Core model:	ruic resului	ig prom.				Core Stage Distribu	tion Scenar	rio Report	ts Chance	-Constrained
7	CP = Purchase cost/unit=	30					2) Specify the stage inf	ormation for	the Variable	es (Adjustał	oles,
8	V=revenue per unit sold=	70					Formulas, Constraints cells), and t	the Random o	cells using t	he set of W	BSP_
9	P=Shortage cost/(unit unsatisfied demand)=	20							Stage:	Refers	
10	H = Holding cost/(unit leftover)=	5					WBSP_RAND WBSP_VAR	•		\$A\$1	4 _
11	Q=Stock level(stage 0 decision)=	87	<<== Stage	e 0 decisio	on.		WBSP_VAR WBSP_RAND		-		
12	D=Demand(stage 1 random variable)=	57.2654	<<== Stage	e 1 randor	n deman	d.	Place function in cell:		 i		
13	LS = Lost sales=	0	<<== Stage	e 1 (recou	rse) deci	sion.	Place function in cell:	\$B\$14			Insert
14	LS >= D - Q (constraint)	>=	<<== Stage	e 1 constra	aint.						
15	I = Inventory	29.7346	<<== Stage	e 1 (recou	rse) deci	sion.					
16	I >= Q - D (constraint)	=>=	<<== Stage	e 1 constra	aint.						
17	TC = Total cost of goods = CP * Q =	2610	<<== Stage	e 0 cost co	omputatio	on.					
18	TH = Total Holding cost=H*I =	148.673	<<== Stage	e 1 compu	te holdin	g cost.	Specifications Optimization Method:			Colum	er Decides 🔻
19	TS = Total Shortage cost= P*LS=	0	<<== Stage	e 1 compu	ite shorta	ge cost.	Seed for Random Gene	rator:			
20	VI = Revenue = V*(D-LS)=	4008.58	<<== Stage	e 1 revenu	e compu	tation.	Common Size per Stage				
21	Profit, expected value, [To be maximized] =										2
22	TP = VI - TC - TH - TS =	1249.903	<<== Stage	e 1 Profit (maximize	e)	Sampling on Continu	ious Distribut	tion Only		
23							Help	<u>C</u> a	ancel		<u>O</u> K



Input via a Dialog Box, Newsvendor, Distribution

J10 =WBSP VAR(0,B11) С D E В FGH J K L Μ Ν Stage 1, in the beginning, unknown demand is revealed to us, and finally in 4 5 Stage 1, at the end, we compute our sales and the resulting profit. 6 1) Core model: 30 CP = Purchase cost/unit= 7 70 V=revenue per unit sold= Add stochastic data here 8 H = Holding cost/(unit leftover)= 5 2) Stage information 9 P=Shortage cost/(unit unsatisfied demand)= 20 WBSP VAR (Q is a stage 0 decision) 10 <<== Stage 0 decision. Q=Stock level(stage 0 decision)= 87 3) Distribution information 11 <<== Stage 1 random demand. WBSP_RAND WBSP_DIST_NORMAL D=Demand(stage 1 random variable)= 57.2654 12 80 Mean demand 13 LS = Lost X Stochastic Support WBSP_VAR 20 S.D. $LS \ge D - Q$ (cor 14 Use Simulation Format ✓ Use Stochastic Modeling 15 Core Stage Distribution Scenario Reports Chance-Constrained 4) Sample size Random # Seed |>= Q - D (cor 16 WBSP_STSC TC = Total cost of goods = 0 17 3) Specify the associated distributions or correlations using the set of 55555 WBSP functions. TH = Total Holding cos 200 1 18 Refers to: TS = Total Shortage cost= 19 B12 WBSP DIST NORMAL - $VI = Revenue = V^*$ 20 WBSP DIST DISCRETE SV W 5) Reporting cells 21 Profit, expected value, To be maxim WBSP_DIST_BETA -WBSP_DIST_BETABINOMIAL WBSP REP 22 TP = VI - TC - THReport on various variables by scenario. WBSP DIST BINOMIAL WBSP_DIST_CAUCHY Insert WBSP HIST Give histogram(s) of variable(s). 23 WBSP_DIST_CHISQUARE WBSP_DIST_EXPONENTIAL 24 WBSP_DIST_F_DISTRIBUTION 25 WBSP_DIST_GAMMA 26 WBSP_DIST_GEOMETRIC WBSP DIST GUMBEL WB! Status WB!_Histogram WBSP DIST HYPERGEOMETRIC WBSP DIST LAPLACE READY WBSP DIST LOGARITHMIC WBSP_DIST_LOGISTIC Solver Decides -WBSP DIST LOGNORMAL WBSP DIST NEGATIVEBINOMIA Model!\$M\$17 WRSP DIST PARETO

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Setting Retention:

Any settings made with a dialog box are retained when the workbook is saved. The same settings will be there when the workbook is next re-opened.

Settings such as Adjustable cells, constraints can be found by clicking on: Add-Ins | WB! | Locate

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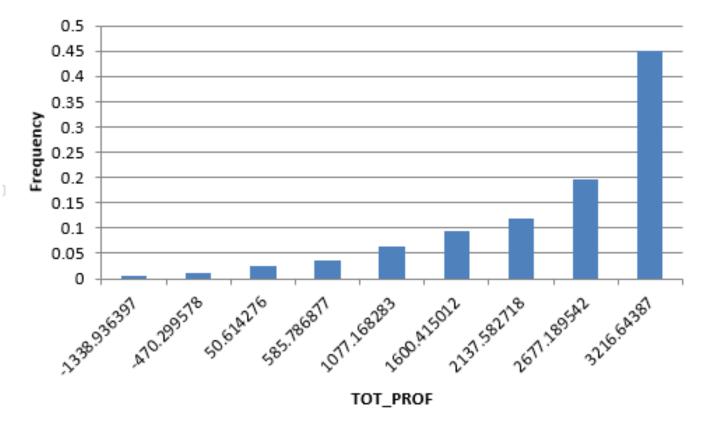
Standard Scenario Report, One Line/Scenario

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19	Expected	Value (E	∀)				L	2485.56			
20	-	Value of				-	-	3189.24			
21	Expected	Value of	Perfec	t Infor	mation (= EVWS-E	V)	703.68			
22	Expected	Value of	Modeli	ng Unce	ertainty	(= EV-EV	EM	41.55	What	does the dist	tributio
4			1	REPORTI	NG CELLS				of To	tal Profit lo	ok like
5	SCENARIO	PROBABI							<i>oj ±0</i>		
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2	- 3-	0	.005		87	102.554	536	15.554636	0	3168.907274	
3	- 4-	0	.005		87	91.376	783	4.376783	0	3392.464335	
4	- 5-	0	.005		87	96.655	923	9.655923	0	3286.88155	
5	- 6-	0	.005		87	45.670	668	0	41.329332	380.300106	
6	- 7-	0	.005		87	103.812	357	16.812857	0	3143.742851	
7	- 8-	0	.005		87	83.128	316	0	3.871684	3189.623731	
8	- 9-	0	.005		87	78.499	431	0	8.500569	2842.457293	
9	- 10-	0	.005		87	98.565	538	11.565638	0	3248.687245	
0	- 11-	0	.005		87	85.013	049	0	1.986951	3330.978684	
1	- 12-	0	.005		87	69.487	583	0		2166.57623	
2	- 13-	0	.005		87	99.691	366	12.691866	0	3226.16268	
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Newsvendor with Normal Demand

Histogram

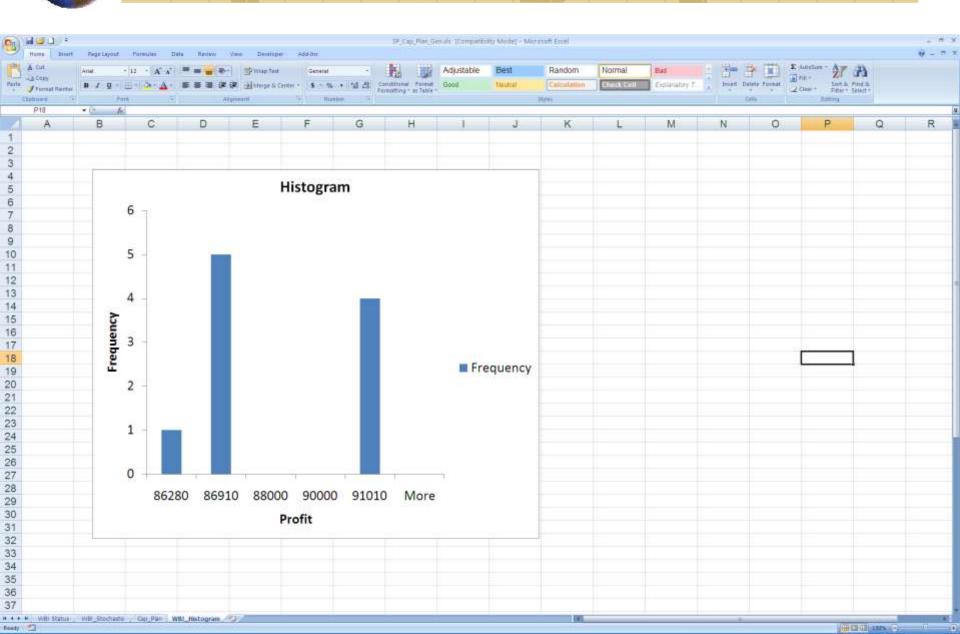


Even though the driving random variable, Demand, has a symmetric distribution, why is the output, Profit, so skewed?

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The Generic Capacity Planning Under Uncertainty Model H (2) 3F Can Plan Genuls [Comparinity Mode] - Microsoft Escel Hotes Page Leyout Adjustable Random Normal Privat Test General Hatterpe & Center * \$ * % * 50 45 Constitional Parent Check Cell Intert Dates Format. Explanatory T. J Format Painter Clear WBSP, RAND(1 823 023) N O P (The Generic) Capacity Planning under Uncertainty Stage 0: We decide what capacities to install at various supply places(inventories, technologies, etc.). Stage 1, Beginning: Demands at various demand locations are revealed, Stage 1, End: We satisfy demands from available capacities (by solving a transportation problem). Step 1: Core Model Cost/unit Capacity Step 2a: Staging info Upper Product to install installed WBSP VAR Declare stage 0 decisions limit Anita 80 300 9999 WBSP RAND Declare stage 1 random variables <= WBSP_VAR Declare stage 1 decisions Daphne 90 383 9999 <= Electra 65 400 <= 9999 Step 2b: Distributions WBSP_DIST_DISCRETE_SV_W Declare discrete joint distribution Generic backup 5 150 =<= 150 10 Total capacity cost: **Demand scenarios** Probability 85220 11 Scenarios Anita Daphne Electra Wgts 12 Variant of Sport Obermeyer, Accurate Response Problem: 300 400 400 0.5 Demand Points 2 333 383 433 0.4 14. Anita Daphne Electra Cannot exceed 500 300 600 0.1 15 Amount shipped **Total capacity** Step 3: Scenario/sampling info 16 300 WBSP_STSC Anita 300 =<= 0 0 Daphne 0 383 0 383 =<= Scenarios Stage 18 Electra 0 0 400 400 =<= 1 10 19 66 Step 4: Reporting info Generic backup 33 0 33 <= 20'WBSP_REP Total in: 333 383 433 Reporting cells (optional): 21 Sales <= Demand: =<= =<= =<= Demands(random): 333 383 433 25 24 Incremental profit/unit Sales revenue Net profit 180 Anita 0 0 176230 91010 <<== Max Daphne 0 160 0 0 Electra 0 140 Generic backup 90 50 60

Capacity Planning Under Uncertainty, Scenario Profit



Capacity Planning, Scenario by Scenario Report

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Plant Location with Random Demand

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	/ 🛅		3.5 X V	orean										90% 🗩			•

The output tab,

WB!_Stochastic, contains two types of information:

- 1) Various expected values that measure the cost of uncertainty,
- 2) A scenario by scenario listing of selected variables so we can explicitly verify what happens in each possible scenario.

We may optionally also generate histograms in a WB!_Histogram tab.

Later, we will discuss the various expected values and the various costs of uncertainty.



Plant Location, Scenario Report

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16	E	xpecte	d Value using E	Expected Valu	ae Policy (EVEVP)	2.666000e+	002							
17	E	xpecte	d Value of Perf	ect Informat	(= EVW	S−EV)	6.000000e-	001							
18	E	xpecte	d Value of Mode	eling Uncerta	ainty (= EV	-EVEVP)	0.000000e+	000							
19															
20				REPORTING CE	ILS										
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Multi-Stage Portfolio Model with Downside Risk

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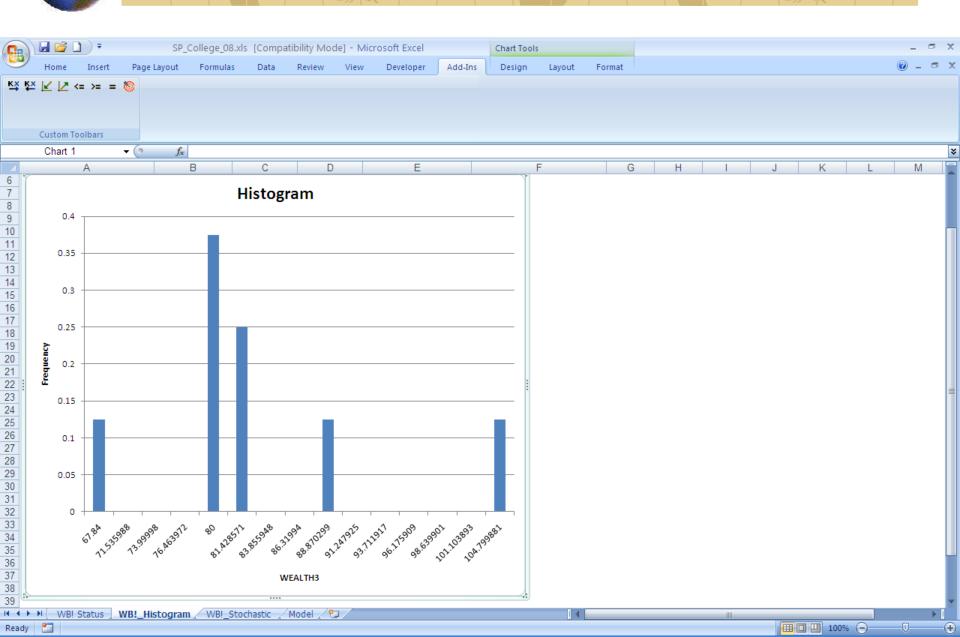
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Multi-stage Portfolio: Solution and Policy

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Terminal Wealth Distribution: College/Retirement Planning



Yield Management: Bird in Hand vs. Future Bird in Bush

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		Random)		ecision)		anosti	-		Price					Specif	y stage			n variable						
8	0	Demand	(5	Sales	20				per seat	Rever	ue				ept decis	ions		/e stage		Dist	ribution	inform	nation	-
9	1	contract of the further on the fact last	=>=	10		>=	0		12		120			WBSP \			WBSP R				IST_DISCRE			
10	2	7'		5	5'		0		15		75			WBSP_V			WBSP_R		WE	BSP_D	IST_DISCRE	TE		
11	3	7		1	4		0		20		20			WBSP_V			WBSP_R	a design of the second s			IST_DISCRE			
12	4	10	>=	1	3'	>=	0		25		25			WBSP_V	AR		WBSP_R	AND	WE	BSP_D	IST_DISCRE	TE		_
13						723		3 1			202								229	12.22	10.00			
14						Max	exp	ected	revenue:		240			WBSP_V	AR						ition of	da		
15 16																			20	ISSID	e deman	us		
														() Sar	nple size					2				
17														WBSP_S						7				
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24.	N WBI Sta	way WBF Stock	astic Co	ore_Hodel	27										11	_	1					Call of the local division of the local divi		10
Ready	12			Contract of the																	(III)	130%	Θ	- 0



Markowitz Portfolio with Min Buy/Cardinality Constraints

Budget constraint, Stage 0; Expected return of portfolio, Stage 1;

! If buy any, must buy at least L_i ;

! Cardinality constraint;

Q is a Positive Semi-Definite *n* by *n* matrix of the covariances of *n* assets. μ_i = expected return of asset *i* during the investment period,

- $\rho = \text{target expected return,}$
- L_i = minimum bought of asset *i*, if any of it is bought,
- U_i = maximum quantity e.g., 1, that can be bought of asset *i*.
- K = upper limit on number assets in portfolio.

This would be an easy convex quadratic problem if it were not for the complicating constraints. LINDO API 9 has much improved methods for finding good solutions quickly to problems of the above type.



Below are some results from letting LINDO API 8 and LINDO API 9 run for at most 300 seconds on a set of problems of the above type. Each problem had from 20 to 400 assets, as indicated in the Problem name.

		Best Results in 300) seconds.		API 9
I	Best known	API 8/LINGO14	API 9/1	LINGO15	Time (sec)
Problem	solution	Best soln	Bound	Best soln	to best
Portdiagcard2	0 0.11022	0.11022	0.11022	0.11022	2
or1200-005-a	66.94	93.76	66.53	66.94	25
or1200-05-c	49.14	54.34	47.14	58.85	10
or1200-05-f	53.35	55.66	50.97	53.88	151
or1300_005_b	98.71	126.06	96.13	98.71	47
or1300_05_e	76.26	94.88	74.32	76.43	158
or1400_05_d	98.89	110.53	96.93	99.49	147
pard200_a	186.00	298.88	184.51	186.00	33
pard300_h	268.86	327.02	265.63	269.85	50
pard400_d	356.90	610.67	355.18	356.90	121
pard400_j	357.45	529.31	352.94	357.44	152

