How to Add Optimization to Planning Under Uncertainty

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General Setting

We have a Planning Model with Optimization:

Maximize some objective, e.g.,
selling_price*volume – production_cost/unit * volume;

subject to various constraints, e.g.,
production at each source ≤ capacity at that source,
supply to each region ≥ demand at that region;

We are not sure of the values of various coefficients in the model, e.g., selling price per unit, cost of raw materials, demand, etc.

What should/can we do? What convenient tools are available, especially in LINGO and What's*Best*! ?

Approaches: Simple to Fancy

Methods that require no additional information beyond the original model:

- Range analysis and dual prices: Works only for Linear Program Models.
- Parametric analysis: Try a range of values for each uncertain parameter.
- *K*-Best solutions: Solve for the *K* best solutions. Which seems most realistic?

Methods that require only Scenario value information for uncertain parameters

- Tornado diagrams: Which parameter uncertainties have biggest effect?
- Fuzzy optimization: Analyze all possible outcomes.
- Robust optimization: Worry about worst possible outcome.
- Data tables in Excel: Automatically generate all outcomes for 1 or 2 parameters.
- Scenario feature of Excel: Enumerate possible scenarios for up to 32 parameters.

Methods that require a distribution of uncertain parameters

- Chance Constrained Programs: Robust optimization with probabilities.
- Value at Risk, Conditional Value at Risk: How bad is the 5% risk case?
- Stochastic Optimization: What is best way to hedge/prepare all possibilities?

Measuring the cost of uncertainty

- Value of More Accurate Forecasts, Can't eliminate variability, but we can know it.
- Value of Modeling Uncertainty, Given the available forecast quality.

Example 1: Product Mix

a) Deterministic Case:

```
! The objective is to maximize profit;
 MAX = 20*ASTRO + 30*COSMO;
          ASTRO    <= 60;  ! Astro line capacity;
             COSMO <= 50;  ! Cosmo line capacity;
! Labor usage <= labor available;
        1*ASTRO + 2*COSMO <= 120;
```

b) Parametric/Uncertain/Scenario Case:

```
 MAX = PAM(1)*ASTRO + PAM(2)*COSMO;
         ASTRO <= PAM(3);  ! Astro demand;
         COSMO <= PAM(4);  ! Cosmo demand;
       PAM(6)*ASTRO + PAM(7)*COSMO <= PAM(5);
```

1) APRFT 2) CPRFT 3) ALCAP 4) CLCAP
5) LABORAVAIL 6) ALBRUSE 7) CLBRUSE;

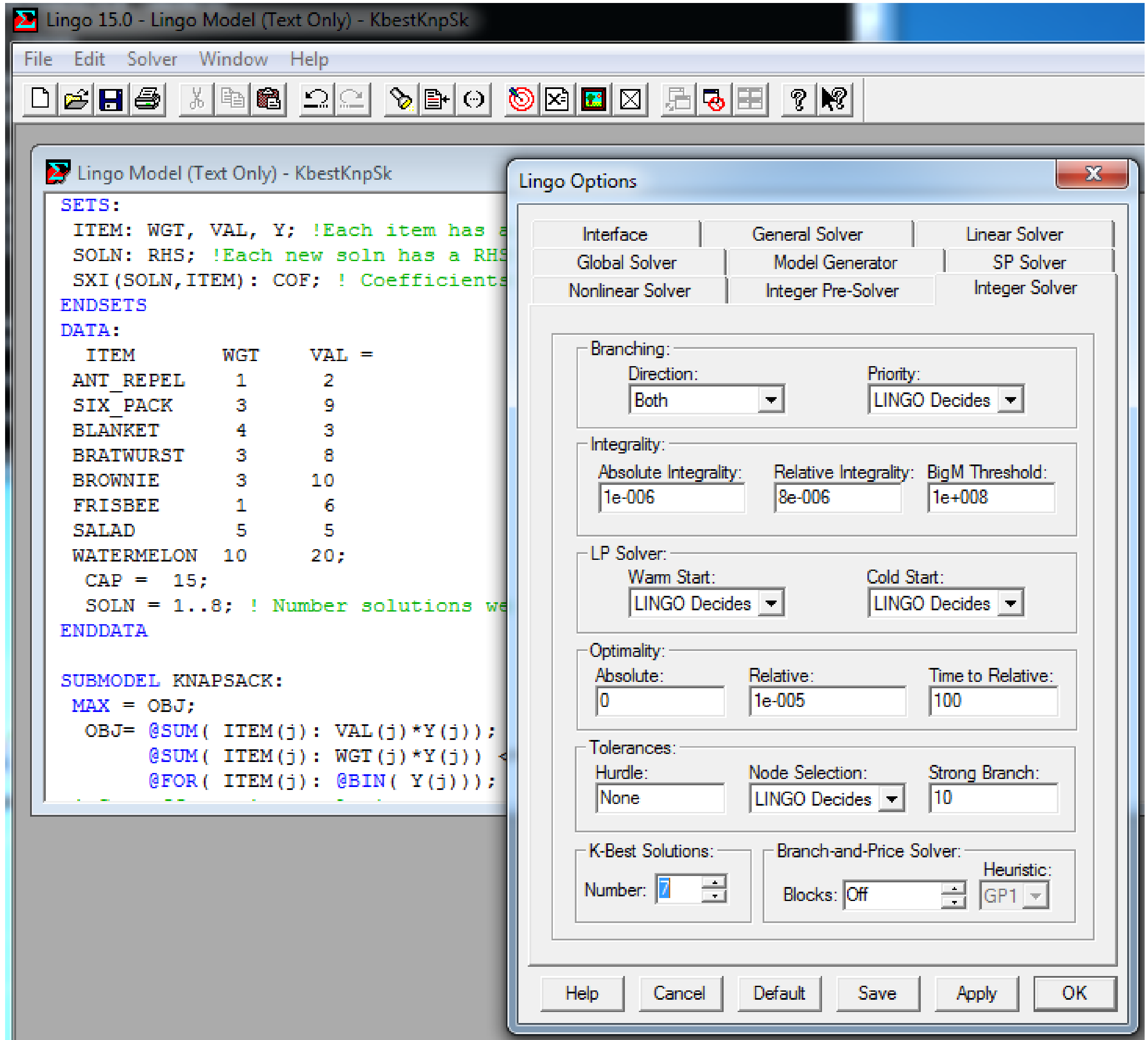
Dual Prices and Range Analysis of an LP

```
 Global optimal solution found.
  Objective value:              2100.000

   Variable           Value        Reduced Cost
      ASTRO        60.00000            0.000000
      COSMO        30.00000            0.000000

        Row    Slack or Surplus      Dual Price
          1        2100.000            1.000000
          2        0.000000            5.000000  ! More Astro Line capacity is worth $5/unit;
          3        20.00000            0.000000  ! More Cosmo Line capacity is worth $0/unit
          4        0.000000            15.00000  ! More Labor capacity is worth $15/unit

! Click on:  Solver -> Range
Ranges in which the basis is unchanged:

                     Objective Coefficient Ranges:
                      Current        Allowable        Allowable
   Variable       Coefficient         Increase         Decrease
      ASTRO          20.00000         INFINITY         5.000000
      COSMO          30.00000         10.00000         30.00000

                         Righthand Side Ranges:
                      Current        Allowable        Allowable
        Row               RHS         Increase         Decrease
          2          60.00000         60.00000         40.00000
          3          50.00000         INFINITY         20.00000
          4          120.0000         40.00000         60.00000
```

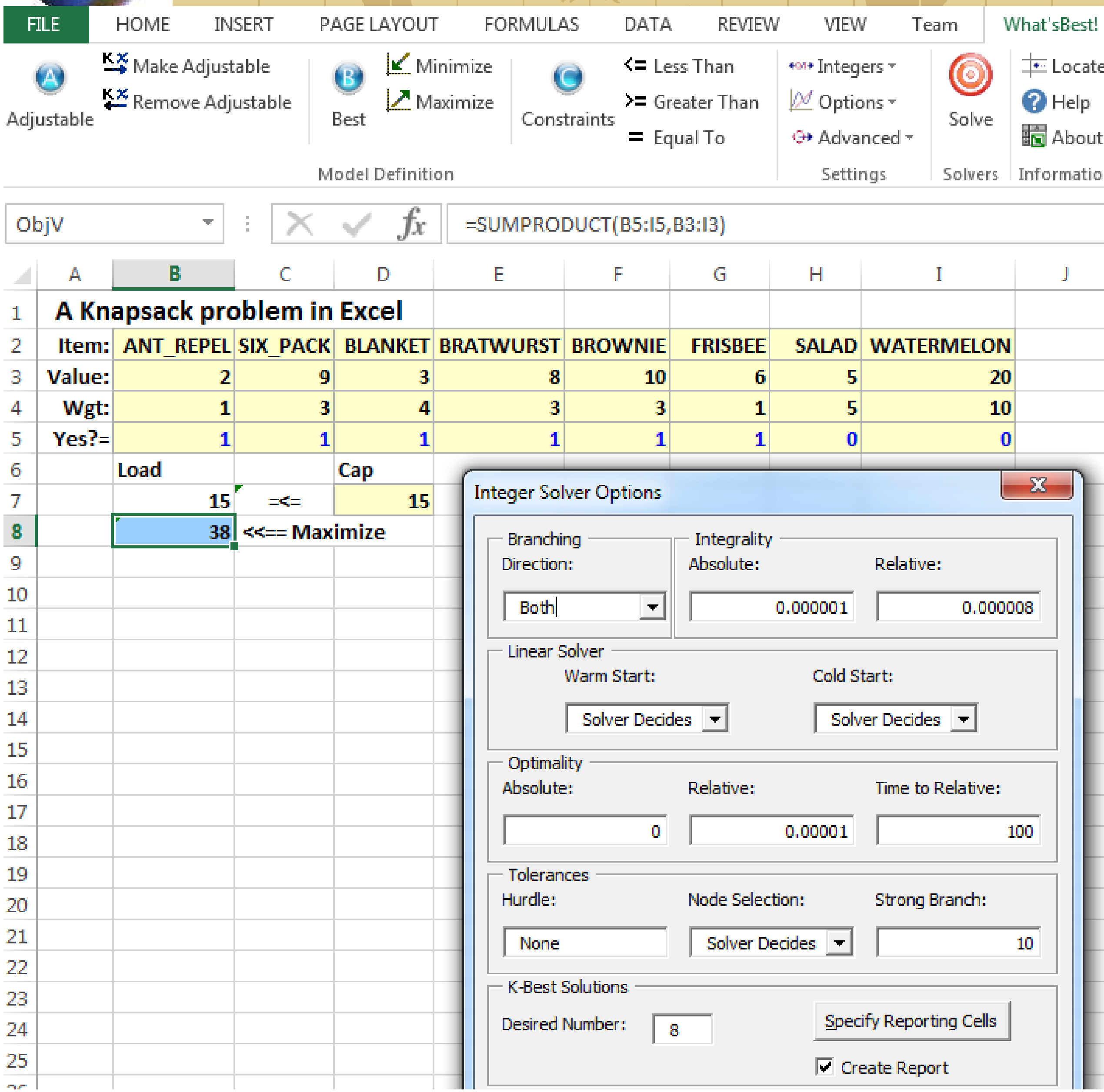
K Best Solutions

```
SETS:
 ITEM: WGT, VAL, Y;   !Each item has a wgt, value, yes/no var;
 ENDSETS
DATA:
  ITEM        WGT     VAL =
 ANT_REPEL     1       2
 SIX_PACK      3       9
 BLANKET       4       3
 BRATWURST     3       8
 BROWNIE       3      10
 FRISBEE       1       6
 SALAD         5       5
 WATERMELON   10      20;
  CAP =   15;
 ENDDATA

MAX = OBJ;
  OBJ= @SUM( ITEM(j): VAL(j)*Y(j));        ! Objective;
       @SUM( ITEM(j): WGT(j)*Y(j)) <= CAP;! Wgt constraint;
       @FOR( ITEM(j): @BIN( Y(j)));        ! All vars 0/1;
```

K Best Solutions in LINGO



K Best Solutions in What'sBest!



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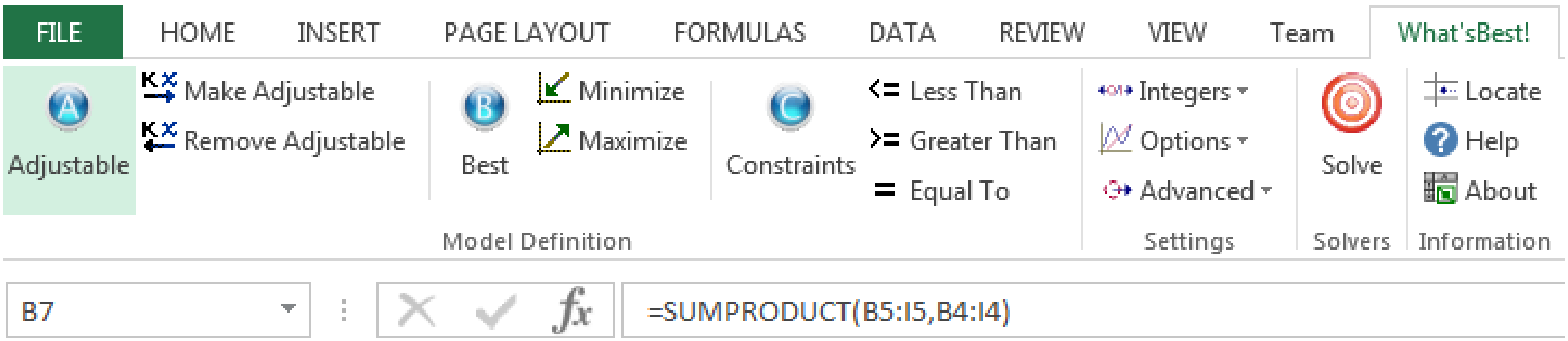
K Best Solutions in What'sBest!



	A	B	C	D	E	F	G	H	I	J
1	**A Knapsack problem in Excel**									
2	Item:	ANT_REPEL	SIX_PACK	BLANKET	BRATWURST	BROWNIE	FRISBEE	SALAD	WATERMELON	
3	Value:	2	9	3	8	10	6	5	20	
4	Wgt:	1	3	4	3	3	1	5	10	
5	Yes?=	1	1	1	1	1	1	0	0	
6		Load		Cap						
7		15	=<=	15						
8		38	<<== Maximize							
9										

K Best Solutions in What'sBest!

	A	B	C	D	E	F	G	H	I	J
5										
6	K-Best Solution Displayed in Spreadsheet: 1									
7										
8		REPORTING CELLS								
9	Run									
10		**ObjVal**	**ANT_REPEL**	**SIX_PACK**	**BLANKET**	**BRATWURST**	**BROWNIE**	**FRISBEE**	**SALAD**	**WATERMELON**
11	----------									
12	- 1-	38	1	1	1	1	1	1	0	0
13	- 2-	38	0	1	0	1	1	1	1	0
14	- 3-	38	1	0	0	0	1	1	0	1
15	- 4-	37	1	1	0	0	0	1	0	1
16	- 5-	36	1	0	0	1	0	1	0	1
17	- 6-	36	0	0	0	0	1	1	0	1
18	- 7-	36	0	1	1	1	1	1	0	0
19	- 8-	35	1	1	0	1	1	1	0	0
20										
21										
22	End of Report									

Parametric Analysis: Markowitz Portfolio

Efficient Frontier Portfolio Calculation (See PortEfFront9a.lng)

The possible investments:

CD___= risk-free rate,
VG040= SP500 stock index,
VG058= Insured long term tax exempt,
VG072= Pacific stock index
VG079= European Stock index,
VG102= Tax managed cap appreciation,
VG533= Emerging markets.

After tax

Target	Risk(1 sd)	Portfolio composition						
Return	1-Yr	CD___	VG040	VG102	VG058	VG079	VG072	VG533
0.04000	0.0000	1.0000						
0.04500	0.0106	0.6483	0.0085	0.0265	0.2496	0.0413	0.0257	
0.05000	0.0212	0.2967	0.0170	0.0530	0.4993	0.0827	0.0513	
0.05500	0.0321		0.1052		0.6645	0.1316	0.0987	
0.06000	0.0541		0.0282		0.5349	0.0926	0.1999	0.1443
0.06500	0.0806				0.4046		0.2863	0.3091
0.07000	0.1087				0.2155		0.3535	0.4310
0.07500	0.1376				0.0264		0.4207	0.5528
0.08000	0.1733							1.0000

Input Data Used:
Expected ret/yr: 0.0400 0.0600 0.0600 0.0500 0.0650 0.0700 0.0800
Stdev in ret/yr: 0.0000 0.0811 0.0911 0.0370 0.1010 0.1252 0.1733

Parametric Analysis


```
! Graph it as is done by Finance folks;
  @CHARTCURVE( 'Return vs Risk', 'Return', 'Risk', 'Standard Deviation', VOUT, VINP);
```

Tornado Diagram Analysis

Recall: Parametric/Uncertain/Scenario Case:

```
  MAX = PAM(1)*ASTRO + PAM(2)*COSMO;
           ASTRO <= PAM(3);  ! Astro demand;
           COSMO <= PAM(4);  ! Cosmo demand;
         PAM(6)*ASTRO + PAM(7)*COSMO <= PAM(5);
```

```
DATA:
 ! Names of the parameters;
  PSET =
 !     1      2    3       4       5      6       7  ;
     APRFT  CPRFT ALCAP  CLCAP LABORAVAIL ALBRUSE CLBRUSE;
 ! The median or base case values for the parameters;
  PMED =
      20     30    60     50    1          2        120;
 ! Plausible low values for the parameters;
  PLO =
      17     25    55     45   0.8        1.7       110;
 ! Plausible high values for the parameters;
  PHI =
      25     38    65     60   1.4         2.3      140;
! For this parameter set we will see that LABORAVAIL has
 the greatest effect on bottom line uncertainty.
 CLCAP has the least effect (none) on bottom line
uncertainty;
ENDDATA
```

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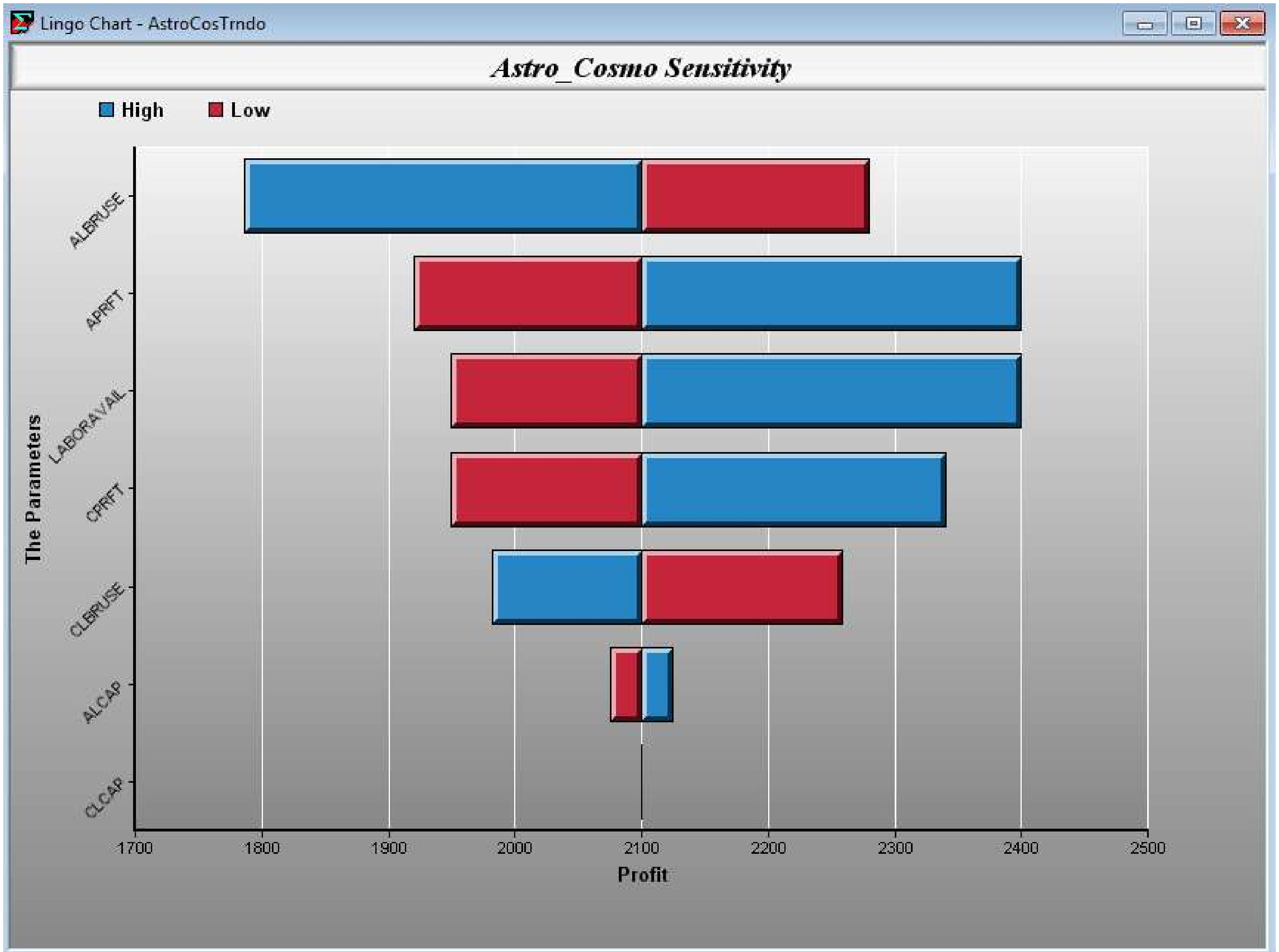
Tornado Diagram Analysis

```
@CHARTTORNADO('Astro_Cosmo Sensitivity', 'Profit', 'The Parameters', BASE, 'High', RESULTHI, 'Low', RESULTLO);
```



Fuzzy Optimization

Given the ranges on the input parameters, what is the range on the optimal objective value? Enumerate all combinations and record.

```
Sensitivity Analysis for an Optimization Problem
with Various Lo and Hi/Fuzzy Parameter Values:
Iter    APRFT    CPRFT    ALCAP    CLCAP  LABRAVL  ALBRUSE  CLBRUSE        Profit
   1    17.00    25.00    55.00    45.00   110.00     0.80     1.70       1905.59
   2    17.00    25.00    55.00    45.00   110.00     0.80     2.30       1652.39
   3    17.00    25.00    55.00    45.00   110.00     1.40     1.70       1531.79
   4    17.00    25.00    55.00    45.00   110.00     1.40     2.30       1293.70
   5    17.00    25.00    55.00    45.00   140.00     0.80     1.70       2060.00
. . .

 124    25.00    38.00    65.00    60.00   110.00     1.40     2.30       1938.91
 125    25.00    38.00    65.00    60.00   140.00     0.80     1.70       3592.06
 126    25.00    38.00    65.00    60.00   140.00     0.80     2.30       3078.91
 127    25.00    38.00    65.00    60.00   140.00     1.40     1.70       2958.57
 128    25.00    38.00    65.00    60.00   140.00     1.40     2.30       2434.57

 The optimal value falls in the range:
[ProfitLo, ProfitHi] = [    1293.6957,     3592.0588]
  Iteration=                      4                 125
```

Data Tables in Excel

PAYMENT | =-PMT(E5,F5,D5)

	A	B	C	D	E	F	G	H
1	**Illustration of a Two-Dimensional Data Table in Excel**							
2	to compute the monthly mortgage payment for various interest rate and term scenarios.							
3		(Col input)	(Row input)	Principal	Monthly	Number of	Monthly	Total
4		Yearly rate	Years	Amount	rate	months	Payment	payments
5		0.06	15	200000	0.005	180	1687.71	25315.70
6								
7								
8			Scenario table of Monthly Payment as a function of Yearly rate and Years					
9		Output cell		Years				
10		**$1,687.71**	**1**	**5**	**10**	**30**		
11	Yearly	**0.02**	16847.773	3505.55	1840.269	739.238945		
12	interest	**0.03**	16938.74	3593.74	1931.215	843.208067		
13	rate	**0.04**	17029.981	3683.3	2024.903	954.830591		
14		**0.05**	17121.496	3774.25	2121.31	1073.64325		
15		**0.06**	17213.286	3866.56	2220.41	1199.10105		
16		**0.07**	17305.349	3960.24	2322.17	1330.60499		
17		**0.08**	17397.686	4055.28	2426.552	1467.52915		
18		**0.09**	17490.295	4151.67	2533.515	1609.24523		
19		**0.1**	17583.177	4249.41	2643.015	1755.14314		
20								
21	Steps:							
22	1) Construct the core/single scenario model. In this example, just row 5.							
23	2) Enter the scenario data, B11:B19, and C10:F10							
24	3) Do a Copy and Paste-Link of G5 to B10 to identify the output cell.							
25	4) Highlight the table B10:F19							
26	5) Click on DATA -> What-if-analysis -> Data Table, and							
27	6) Enter C5 as the Row Input cell, and							
28	B5 as the Column Input cell.							
29								

Scenario Manager in Excel

F5 =YEARS*12

	A	B	C	D	E	F	G	H
1	**Illustration of Scenario Manager in Excel**							
2	to compute the monthly mortgage payment for various interest rate and term scenarios.							
3				Principal	Monthly	Number of	Monthly	Total payments
4		Yearly rate	Years	amount	rate	months	Payment	payments
5		0.05	25	200000	0.004167	300	1169.18	29229.50

Steps:

1) Construct the core/single scenario model. In this example, just row 5.

2) Click on DATA -> What-if-analysis -> Scenario Manager, to

Select, Display, or Edit a particular Scenario, i.e., values for B5, C5, et

Scenario Manager

Scenarios:

CASE1506
CASE2006
CASE3006
CASE2505

Add...
Delete
Edit...
Merge...
Summary...

Changing cells: $B$5:$C$5

Comment: Created by on 10/24/2015
Modified by on 10/24/2015
Modified by on 10/26/2015

Robust Optimization

Setting:

0) We make a decision, e.g., inventory levels, investments, etc.

1) Nature makes a random decision.

First identify a set of possible scenarios/outcomes for the random variables. In (0),

Choose the decision the maximizes the profit, subject to being feasible for every possible scenario.

Slightly more mathematically:

minimize $f_0(x)$ (or maximize, as desired)

subject to

For every constraint i:

For every scenario s:

$f_i(x, u_s) \geq 0$;

Portfolio Example, Illustrating Robust Optimization

Example : Portfolio investment

a) Deterministic Case:

```
! Maximize end-of-period wealth;
MAX = 1.089*ATT + 1.214*GMC + 1.235*USX + 1.05*XTBILL;
! We have $1M to start;
            ATT +        GMC +        USX +     XTBILL= 1;
```

b) Parametric/Uncertain/Scenario Case:

```
MAX=PAM(1)*ATT + PAM(2)*GMC + PAM(3)*USX + 1.05*XTBILL;
          ATT +        GMC +        USX +      XTBILL = 1;
   PAM(1)*ATT + PAM(2)*GMC + PAM(3)*USX + 1.05*XTBILL >= TARGET;
```

Portfolio Example, Scenarios

```
! Some equally likely scenarios of future values of
each of the instruments per $1 invested today
        ATT      GMC      USX ;
        1.300    1.225    1.149
        1.103    1.290    1.26
        1.216    1.216    1.419
        0.954    0.728    0.922
        0.929    1.144    1.169
        1.056    1.107    0.965
        1.038    1.321    1.133
        1.089    1.305    1.732
        1.090    1.195    1.021
        1.083    1.390    1.131
        1.035    0.928    1.006
        1.176    1.715    1.908;
```

Robust Optimization

```
DATA:
 TBILLGF = 1.05; ! Risk free growth factor, e.g., for money invested in Treasury Bills;
 TARGET = 1.01;  ! Target growth factor;
 SCENE = 1..12;  ! Number of scenarios;
 ! Our investment opportunities, in addition to T Bills;
 ASSET=   ATT      GMC      USX ;
    . . .
ENDDATA

 NS = @SIZE( SCENE); ! Number scenarios;
! Stage 0: Choose the X's and AVG;

! Budget constraint at beginning;
 [BUD] @SUM( ASSET( J): X(J))+ XTBILL = 1;
    @FOR( SCENE( s):
! Compute R(s) =  value of total portfolio under scenario s.
   X(i) = amount invested in instrument i;
      R( s) = @SUM( ASSET( j): GF( s, j) * X( j))+ XTBILL*TBILLGF;
        );

! Compute expected value of ending position,
  assuming all scenarios equally likely;
  AVG = @SUM(SCENE(s): R(s))/ NS;

! Robustness constraints:
    We want to beat the target in every scenario;
    @FOR( SCENE( s):
      R( s) >= TARGET;
        );

! A reasonable objective: Maximize average return;
   MAX = AVG;
```

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Robust Optimization

Variable	Value
AVG	1.078841
TBILLGF	1.050000
TARGET	1.030000
XTBILL	**0.8437500**
X(ATT)	**0.000000**
X(GMC)	**0.000000**
X(USX)	**0.1562500**
R(1)	1.065469
R(2)	1.082813
R(3)	1.107656
R(4)	**1.030000**
R(5)	1.068594
R(6)	1.036719
R(7)	1.062969
R(8)	1.156563
R(9)	1.045469
R(10)	1.062656
R(11)	1.043125
R(12)	1.184063

Recall:

ATT	GMC	USX ;
1.300	1.225	1.149
1.103	1.290	1.26
1.216	1.216	1.419
0.954	0.728	0.922
0.929	**1.144**	**1.169**
1.056	**1.107**	**0.965**
1.038	1.321	1.133
1.089	1.305	1.732
1.090	1.195	1.021
1.083	1.390	1.131
1.035	**0.928**	**1.006**
1.176	1.715	1.908;

Chance Constrained Optimization

Chance Constrained Programing:

we are allowed to violate certain specified constraints with a specified (typically low) probability;

```
! Chance constraints;
   @FOR( SCENE( s):
! ZSAT( s) = 1 if we satisfy constraint in scenario s;
     @BIN( ZSAT(s)); ! It is 0 or 1;
     R( s) >= ZSAT( s) * TARGET;
       );

!    We want to beat the target this fraction of the time ;
   @SUM( SCENE( s): ZSAT(s))/ NS >= PROBCC;

! A reasonable objective: Maximize average return;
  MAX = AVG;
```

Chance Constrained Optimization

Variable	Value
AVG	1.225009
TBILLGF	1.050000
TARGET	1.030000
PROBCC	0.8300000
XTBILL	0.000000
X(ATT)	**0.000000**
X(GMC)	**0.4577465**
X(USX)	**0.5422535**
R(1)	1.183789
R(2)	1.273732
R(3)	1.326077
R(4)	**0.8331972**
R(5)	1.157556
R(6)	**1.030000**
R(7)	1.219056
R(8)	1.536542
R(9)	1.100648
R(10)	1.249556
R(11)	**0.9702958**
R(12)	1.819655
ZSAT(1)	1.000000
ZSAT(2)	1.000000
ZSAT(3)	1.000000
ZSAT(4)	0.000000
ZSAT(5)	1.000000
ZSAT(6)	1.000000
ZSAT(7)	1.000000
ZSAT(8)	1.000000
ZSAT(9)	1.000000
ZSAT(10)	1.000000
ZSAT(11)	0.000000
ZSAT(12)	1.000000

Recall:

```
   ATT      GMC      USX ;
  1.300    1.225    1.149
  1.103    1.290    1.26
  1.216    1.216    1.419
  0.954    0.728    0.922
  0.929    1.144    1.169
  1.056    1.107    0.965
  1.038    1.321    1.133
  1.089    1.305    1.732
  1.090    1.195    1.021
  1.083    1.390    1.131
  1.035    0.928    1.006
  1.176    1.715    1.908;
```

Criterion Choice – Utility Function Choice

What Should Our Objective Criterion be Under Uncertainty?

Desirable Features of a Utility Function:

1) More is better: An additional dollar is always appreciated, no matter how much we have already.
2) Concavity: Twice as much is not twice better. The $(n+1)^{st}$ dollar is no more valuable than the n^{th} dollar.

Value at Risk

To use VaR, you must specify two numbers:
1) a probability threshold, typically 5% (or 1%), beyond which you care about bad outcomes.
2) an interval of time, typically one day or ten days, over which you are concerned about losing money,
VaR = amount of loss in one day that has at most a 5% (or 1%) probability of being exceeded.

VaR is a method recommended as part of the Basel Accord for measuring the risk of the portfolios of European banks. Banks must hold capital reserves proportional to their risk, e.g., as measured by VaR.

Solution:

Variable	Value
TBILLGF	1.050000
RHO	**0.1670000**
NS	12.00000
BIGM	1.180000
XTBILL	**0.000000**
X(ATT)	**0.000000**
X(GMC)	**1.000000**
X(USX)	**0.000000**
AVG	1.213667
T	1.107000
R(1)	1.225000
R(2)	1.290000
R(3)	1.216000
R(4)	**0.7280000**
R(5)	1.144000
R(6)	1.107000
R(7)	1.321000
R(8)	1.305000
R(9)	1.195000
R(10)	1.390000
R(11)	**0.9280000**
R(12)	1.715000

```
! Some equally likely scenarios of the
future values of each of the instruments
per $1 invested today;
 ASSET=    ATT       GMC      USX ;
 GF =    ! Growth Factors, each investment
          1.300    1.225    1.149
          1.103    1.290    1.26
          1.216    1.216    1.419
          0.954    0.728    0.922
          0.929    1.144    1.169
          1.056    1.107    0.965
          1.038    1.321    1.133
          1.089    1.305    1.732
          1.090    1.195    1.021
          1.083    1.390    1.131
          1.035    0.928    1.006
          1.176    1.715    1.908;
```

Conditional Value at Risk

CVaR requires us to specify a risk tolerance ρ, e.g., 5%. If the random variable w is the final wealth of the portfolio, then CVaR chooses a portfolio and VaR threshold, t, so as to maximize a weighted combination of: the final portfolio value, the VaR value, and minus the expected amount by which the final portfolio falls short of the VaR target. Optionally, we may specify an expected return preference $\alpha \geq 0$. Algebraically, the CVaR objective is:

$$\text{Max } \alpha\, \mathrm{E}(w) + \rho\, t - \mathrm{E}(\max[0, t - w]).$$

Conditional Value at Risk, Details

```
! Compute portfolio value, R(s),  under each scenario s;
  @FOR(SCENE(S): R(S) = @SUM(ASSET(J):VE(S,J) * X(J));
! Measure deviations from target T;
     DVL( S) - DVU( S) = T - R(S) ;
      );
! Compute expected value of ending position;
 [DEFAVG] AVG = @SUM( SCENE(s): PRB(s) * R(s));
! Ending value >= target ;
 [RET] AVG >= TARGET;
! Minimize conditional value at risk;
 [OBJV] MAX = OBJ;         OBJ = ALPHA*AVG + RHO*T - @SUM( SCENE(s): PRB(s)* DVL(s));
! Notice that as long as the fraction of the scenarios with
  R(s) < T is < RHO, we ( and the optimizer) can increase T;
```

Variable	Value
RHO	**0.1660000**
TARGET	1.150000
OBJ	0.1625300
T	**1.017901**
AVG	**1.150000**
X(ATT)	**0.5813256**
X(USX)	**0.4186744**
R(1)	1.236780
R(2)	1.168732
R(3)	1.300991
R(4)	**0.9406024**
R(5)	1.029482
R(6)	**1.017901**
R(7)	1.077774
R(8)	1.358208
R(9)	1.061111
R(10)	1.103096
R(11)	1.022858
R(12)	1.482470

```
ASSET=   ATT      GMC      USX ;
GF =    ! Growth Factors, each investment;
        1.300    1.225    1.149
        1.103    1.290    1.26
        1.216    1.216    1.419
        0.954    0.728    0.922
        0.929    1.144    1.169
        1.056    1.107    0.965
        1.038    1.321    1.133
        1.089    1.305    1.732
        1.090    1.195    1.021
        1.083    1.390    1.131
        1.035    0.928    1.006
        1.176    1.715    1.908;
```

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Portfolio's: Various Objectives

```
! Scenario portfolio model with various possible objectives.
 See end of model
 LINGO will automatically choose the appropriate solver:
  Linear, Quadratic/Second Order Cone, or Nonlinear;
SETS:
 SCENE: PROB, R, DVU, DVL, DV2, DV3, DV1;
 ASSET: X;  ! X(j) = amount to invest in asset j;
 SXA( SCENE, ASSET): GF ;
ENDSETS
DATA:
 TBILLGF = 1.05; ! Risk free growth factor, e.g., for money invested in Treasury Bills;
 TARGET = 1.15;  ! Target growth factor;
 SCENE = 1..12;  ! Number of scenarios;
 ! Our investment opportunities, in addition to T Bills;
 ASSET=   ATT     GMC     USX ;
! Some equally likely scenarios of the future
  values of each of the intruments per $1 invested today;
 GF =    ! The yearly Growth Factors for each investment;
        1.300   1.225   1.149
        1.103   1.290   1.260
        1.216   1.216   1.419
        0.954   0.728   0.922
        0.929   1.144   1.169
        1.056   1.107   0.965
        1.038   1.321   1.133
        1.089   1.305   1.732
        1.090   1.195   1.021
        1.083   1.390   1.131
        1.035   0.928   1.006
        1.176   1.715   1.908;
! All scenarios equally likely;
 PROB =.083333  .083333  .083333  .083333  .083333  .083333
       .083333  .083333  .083333  .083333  .083333  .083333;
ENDDATA
```

Portfolios, Various Objectives, Basic Model.

```
 @FREE( AVG);
! Stage 0: Choose the X's and AVG;
! Budget constraint;
 [BUD] @SUM( ASSET( J): X(J))+ XTBILL = 1;
! Target ending value;
 [RET] AVG >= TARGET;

! Stage 1:
 @FOR( SCENE( S):
     @FREE( R( S));
! Compute R(s) =  value of total portfolio under scenario s.
   X(i) = amount invested in instrument i;
   R( s) = @SUM( ASSET( j): GF( s, j) * X( j))+ XTBILL*TBILLGF;
! Measure deviations up and below from average;
   DVU( s) - DVL( s) = R(s) - AVG;
        );
! Compute expected value of ending position;
 AVG = @SUM(SCENE(S): PROB(S) * R(S));
```

Portfolios, Various Objectives, I

```
! Set objective to one of the following...;

! Linear objectives;
!   1) Minimum absolute deviation(MAD) in return;
!   MIN = @SUM( SCENE(s): PROB(s) *( DV1(s)));

!   2) Downside risk;
!  MIN = @SUM( SCENE(s): PROB(s) * DVL(s));

! Quadratic objectives;
!   3) Simple variance;
!   MIN = @SUM( SCENE(s): PROB(s) * ( DV1( s))^2);

!   4) Semi-variance, or squared downside risk;
!   MIN = @SUM( SCENE(s): PROB(s) * DVL(s)^2);

! Conic objective,
    5)Value-at-Risk, assuming Normal Distribution
  and a 5% risk tolerance;
!   SD^2 >= @SUM( SCENE(s): (DV1(s))^2);
! Maximize a weighted combination of mean less SD;
!   MAX = AVG - 1.645*SD;
```

Portfolios, Various Objectives, II

```
! Nonlinear objective;
!   6) Absolute deviations raised to 3rd power ( We hate large deviations);
!    MIN = @SUM( SCENE(s): PROB(s) * ( DV1(s))^3);

!   7) Tell Global solver to trust us that we know the objective is convex;
!   Deviations raised 3rd power ( We hate large deviations);
!   CUROBJ >c= @SUM( SCENE(s): PROB(s) * ( DV1(s))^3);
!   MIN = CUROBJ;

!   8) To illustrate the generality of Conic/SOC capability,
  3rd power objective converted to SOC form;
!   @FOR( SCENE(s):
     DV2(s) >= DV1(s)^2;
!  Force DV3 to = the third power;
!      DV3(s)* DV1(s) >= DV2(s)^2;
!       );
!  Sum of the 3rd powers;
!   MIN = @SUM( SCENE(s): PROB(s)*DV3(s));
!  In fact, any power > 1 can be converted to SOC;
```

Portfolios, Various Objectives,

```
Case 4 (Semi-variance):
 Global optimal solution found.
 Objective value:                                    0.7155100E-02
 Elapsed runtime seconds:                                 0.08
 Model is convex quadratic

     Variable           Value
      X( ATT)       0.1903418E-06
      X( GMC)       0.1068679
      X( USX)       0.4470278
```

Portfolios, Various Objectives

```
Case 6 (NLP, Deviations to 3rd power)
 Local optimal solution found.
 Objective value:                              0.4715426E-02
 Elapsed runtime seconds:                           0.09

           Variable           Value
            X( ATT)       0.3371926
            X( GMC)       0.3272703
            X( USX)       0.1802045
```

Portfolios, Various Objectives

```
Case 8 ( Deviations to 3rd power as SOC):
  Global optimal solution found.
  Objective value:                                    0.4715455E-02
  Elapsed runtime seconds:                                 0.11
  Model is a second-order cone

      Variable           Value
        XTBILL       0.1553326
       X( ATT)       0.3371927
       X( GMC)       0.3272697
       X( USX)       0.1802050
```

The "gold standard" for planning under uncertainty.

How is SP Information Stored in the SpreadSheet?

All information about the SP features is stored explicitly/openly on the spreadsheet.

1) Core model is a regular deterministic What'sBest! or LINGO model. You can plug in regular numbers in a random cell to check results.

2) Staging information is stored in
Decisions: WBSP_VAR(stage, cell_list) and
Random variables: WBSP_RAND(stage, cell_list);

3) Distribution specification is stored in
WBSP_DIST_distn(table, cell_list);
where distn specifies the distribution, e.g., NORMAL cell.

4) Sample size for each stage is stored in
WBSP_STSC(table);

5) Cells to be reported are listed in
WBSP_REP(cell_list) or WBSP_HIST(bins, cell);

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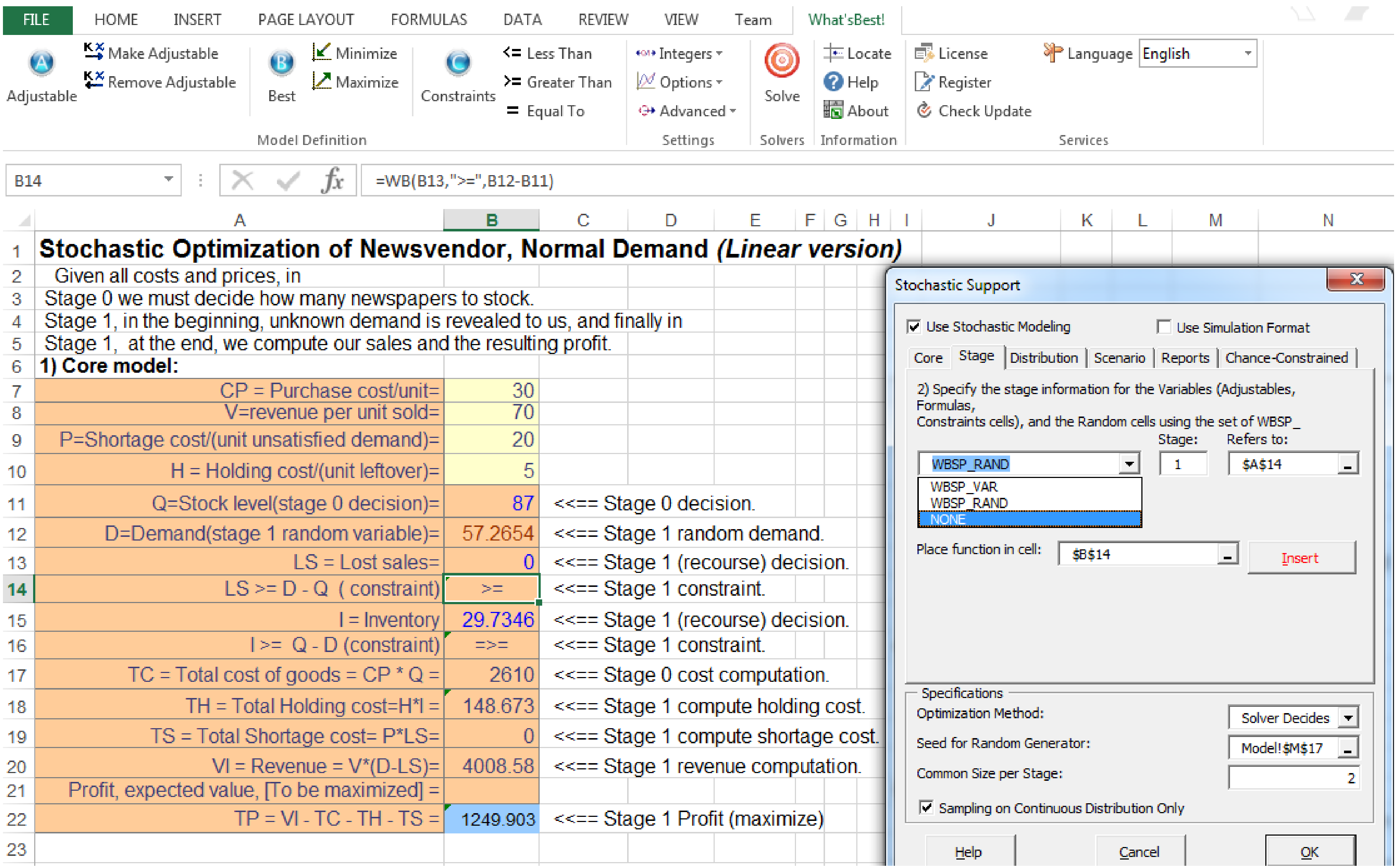
Core Comments

The “Core Model” is a completely valid Excel model.

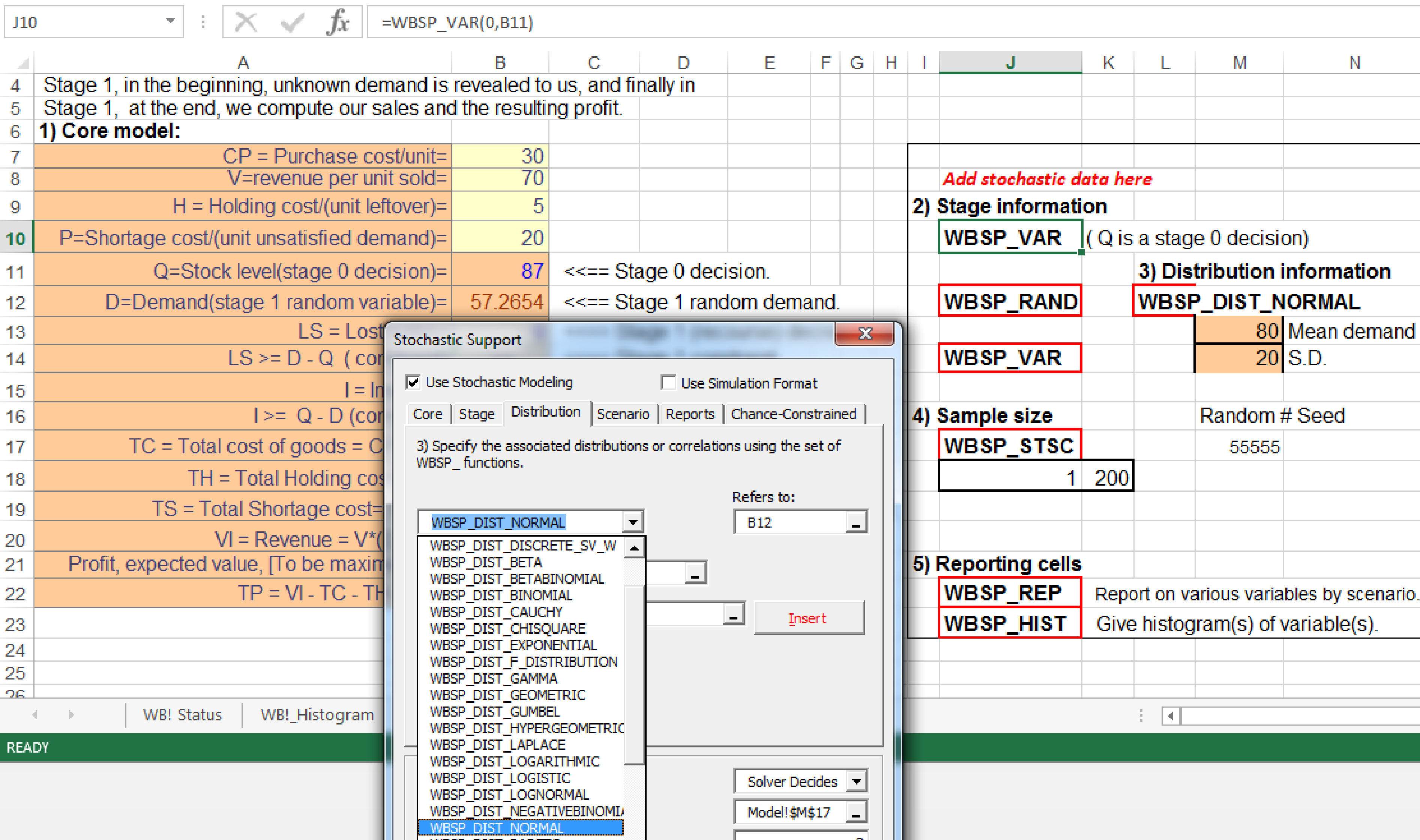
If you are doing neither simple optimization nor SP, you can do complete “What-If” analysis with it as a valid deterministic model.

If you have not turned on SP, you can do simple optimization with it like any deterministic What’sBest model.

Stochastic Optimization: Newsvendor in What'sBest!



Input via a Dialog Box, Newsvendor, Distribution



Input via a Dialog Box, Setting Various Options

Setting Retention:

Any settings made with a dialog box are retained when the workbook is saved. The same settings will be there when the workbook is next re-opened.

Settings such as Adjustable cells, constraints can be found by clicking on:

Add-Ins | WB! | Locate

Standard Scenario Report, One Line/Scenario

E19 | 2485.559

19	Expected Value (EV)	2485.56
20	Expected Value of Wait-and-See Model's Objective (	3189.24
21	Expected Value of Perfect Information (=|EVWS-EV|)	703.68
22	Expected Value of Modeling Uncertainty (=|EV-EVEM|	41.55

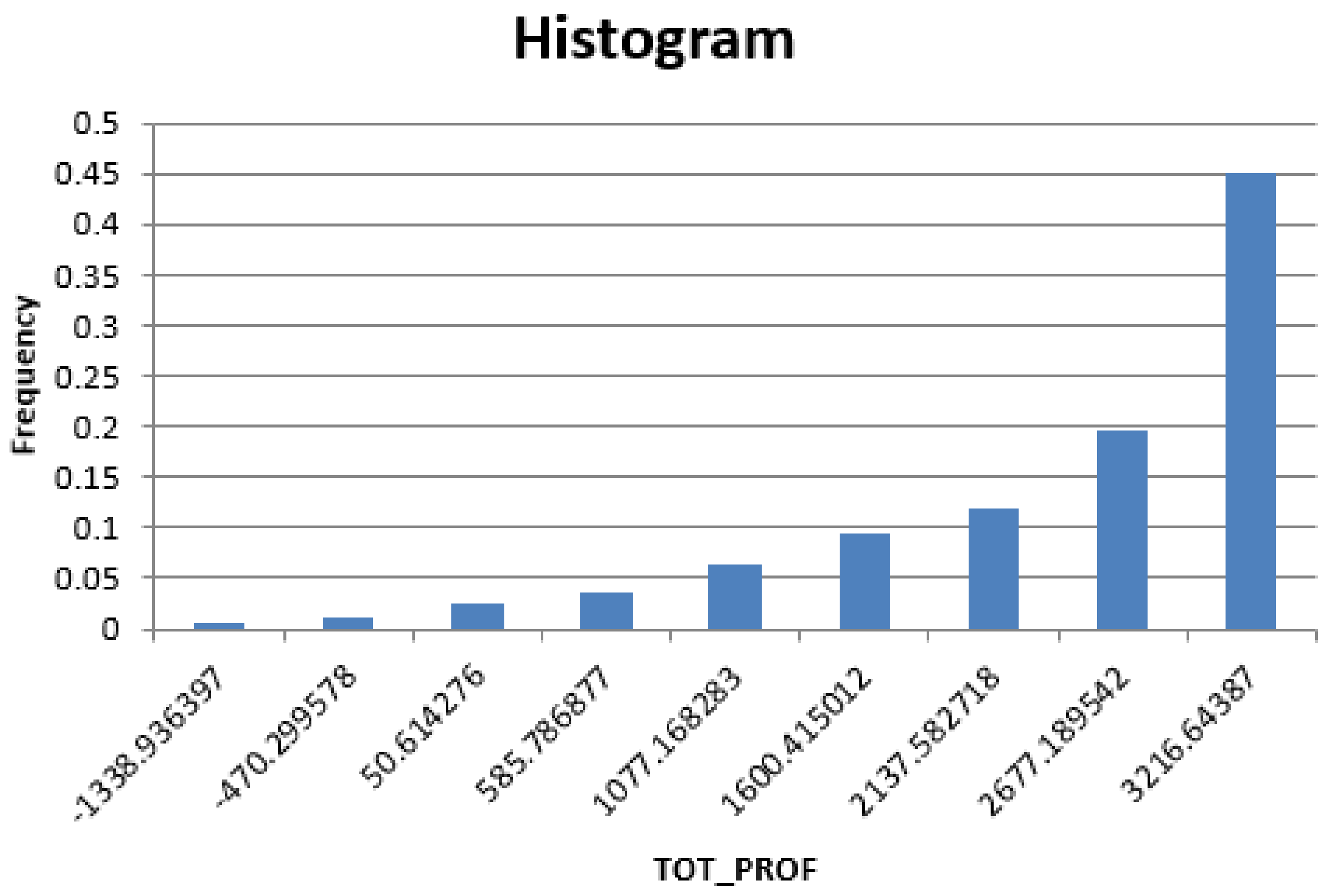
What does the distribution of Total Profit look like?

REPORTING CELLS

SCENARIO	PROBABILITY	Model!B11 Q STAGE 0	Model!B12 D STAGE 1	Model!B13 LOSTSALES STAGE 1	Model!B15 INVENTORY STAGE 1	Model!B22 TOT_PROF STAGE 1
- 1-	0.005	87	57.26538	0	29.73462	1249.903478
- 2-	0.005	87	55.632736	0	31.367264	1127.455202
- 3-	0.005	87	102.554636	15.554636	0	3168.907274
- 4-	0.005	87	91.376783	4.376783	0	3392.464335
- 5-	0.005	87	96.655923	9.655923	0	3286.88155
- 6-	0.005	87	45.670668	0	41.329332	380.300106
- 7-	0.005	87	103.812857	16.812857	0	3143.742851
- 8-	0.005	87	83.128316	0	3.871684	3189.623731
- 9-	0.005	87	78.499431	0	8.500569	2842.457293
- 10-	0.005	87	98.565638	11.565638	0	3248.687245
- 11-	0.005	87	85.013049	0	1.986951	3330.978684
- 12-	0.005	87	69.487683	0	17.512317	2166.57623
- 13-	0.005	87	99.691866	12.691866	0	3226.16268
- 14-	0.005	87	133.747367	46.747367	0	2545.052659
- 15-	0.005	87	43.663515	0	43.336485	229.763642
- 16-	0.005	87	87.908198	0.908198	0	3461.836042

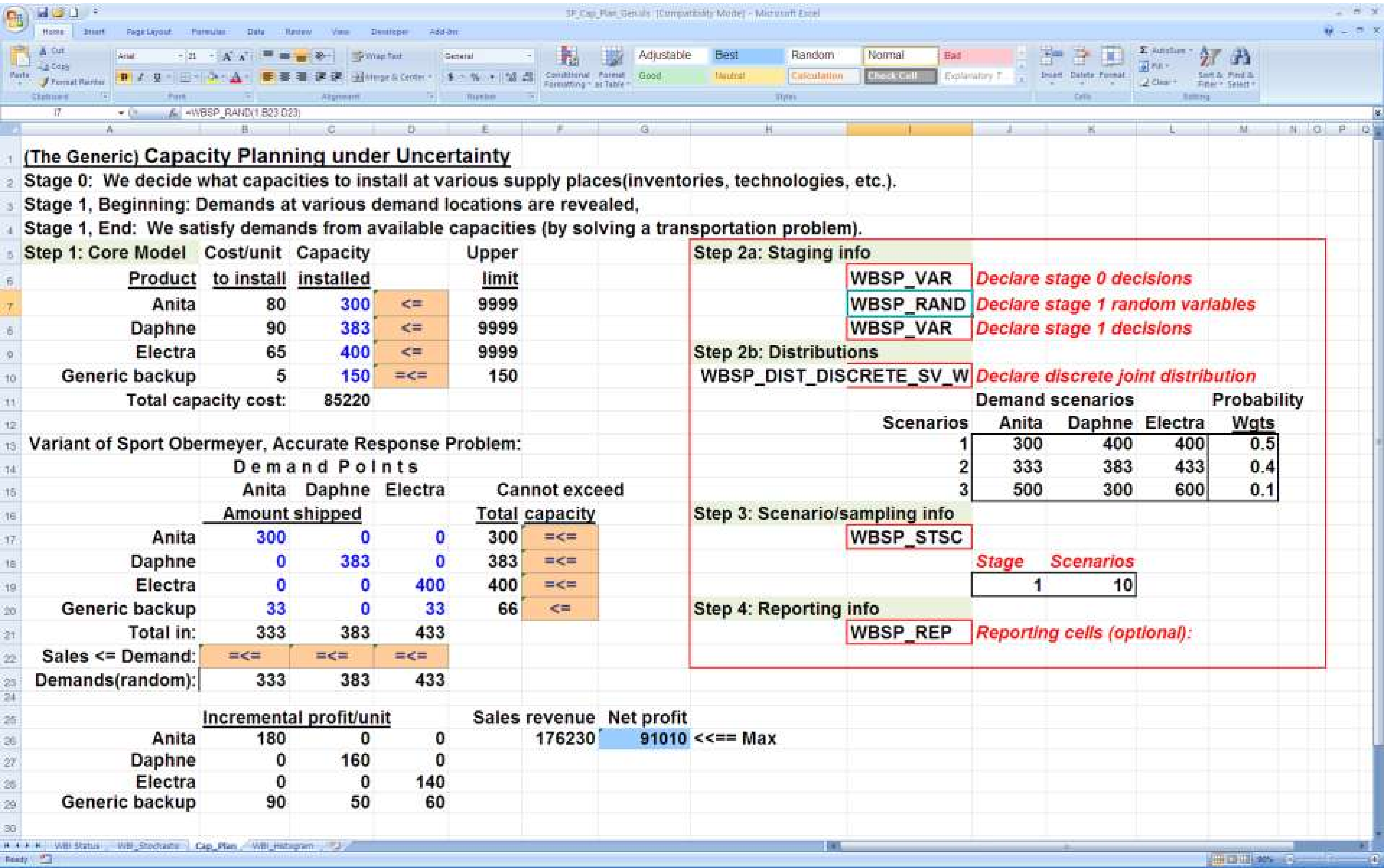
WB! Status | WB!_Histogram | **WB!_Stochastic** | Model

Newsvendor with Normal Demand



Even though the driving random variable, Demand, has a symmetric distribution, why is the output, Profit, so skewed?

The Generic Capacity Planning Under Uncertainty Model



(The Generic) Capacity Planning under Uncertainty

Stage 0: We decide what capacities to install at various supply places(inventories, technologies, etc.).

Stage 1, Beginning: Demands at various demand locations are revealed,

Stage 1, End: We satisfy demands from available capacities (by solving a transportation problem).

Step 1: Core Model Product	Cost/unit to install	Capacity installed		Upper limit
Anita	80	300	<=	9999
Daphne	90	383	<=	9999
Electra	65	400	<=	9999
Generic backup	5	150	=<=	150
Total capacity cost:	85220			

Variant of Sport Obermeyer, Accurate Response Problem:

	Demand Points			Cannot exceed	
	Anita	Daphne	Electra		
	Amount shipped			Total	capacity
Anita	300	0	0	300	=<=
Daphne	0	383	0	383	=<=
Electra	0	0	400	400	=<=
Generic backup	33	0	33	66	<=
Total in:	333	383	433		
Sales <= Demand:	=<=	=<=	=<=		
Demands(random):	333	383	433		

	Incremental profit/unit			Sales revenue	Net profit	
Anita	180	0	0	176230	91010	<<== Max
Daphne	0	160	0			
Electra	0	0	140			
Generic backup	90	50	60			

Step 2a: Staging info

WBSP_VAR	*Declare stage 0 decisions*
WBSP_RAND	*Declare stage 1 random variables*
WBSP_VAR	*Declare stage 1 decisions*

Step 2b: Distributions

WBSP_DIST_DISCRETE_SV_W *Declare discrete joint distribution*

	Demand scenarios			Probability
Scenarios	Anita	Daphne	Electra	Wgts
1	300	400	400	0.5
2	333	383	433	0.4
3	500	300	600	0.1

Step 3: Scenario/sampling info

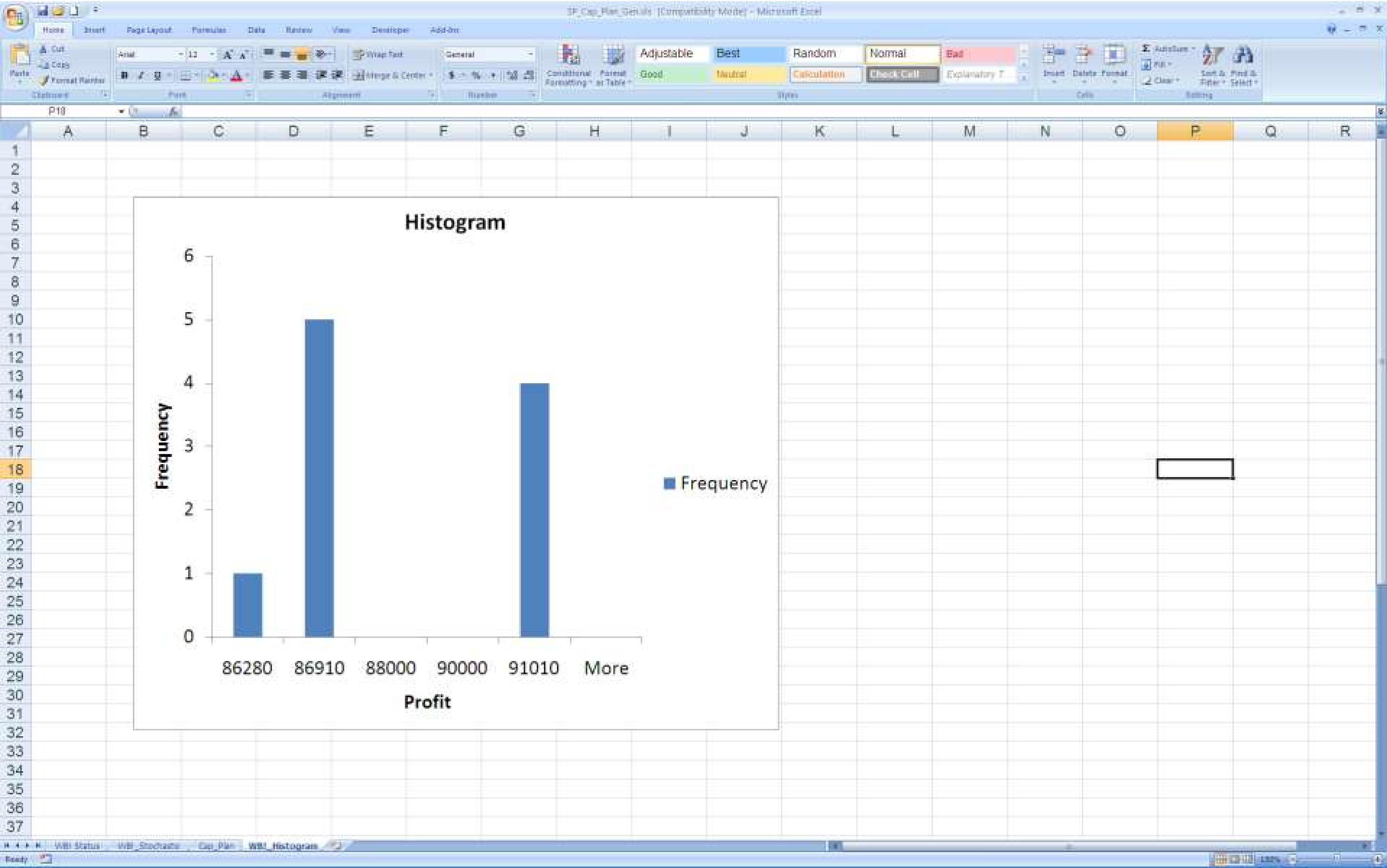
WBSP_STSC

Stage	*Scenarios*
1	10

Step 4: Reporting info

WBSP_REP *Reporting cells (optional):*

Capacity Planning Under Uncertainty, Scenario Profit



Capacity Planning, Scenario by Scenario Report

STOCHASTIC INFORMATION

RANDOMS	3
STAGES	2
NODES	4
SCENARIOS	3

Expected Value (EV)	8.848700e+004
Expected Value of Wait-and-See (EVWS)	9.263400e+004
Expected Value using Expected Value Policy (EVEVP)	8.595240e+004
Expected Value of Perfect Information (=|EVWS-EV|)	4.147000e+003
Expected Value of Modeling Uncertainty (=|EV-EVEVP|)	2.534600e+003

REPORTING CELLS

SCENARIO	PROBABILITY	Cap_Plan!C7 CAPANITA STAGE 0	Cap_Plan!C8 CAPDAPHNE STAGE 0	Cap_Plan!C9 CAPELECTRA STAGE 0	Cap_Plan!C10 CAPGENERIC STAGE 0	Cap_Plan!B23 DEMANITA STAGE 1	Cap_Plan!B20 GENANITA STAGE 1	Cap_Plan!C23 DEMDAPHNE STAGE 1	Cap_Plan!C20 GENDAPHNE STAGE 1	Cap_Plan!D23 DEMELECTRA STAGE 1	Cap_Plan!D20 GENELECTRA STAGE 1	Cap_Plan!G26 WBMAX STAGE 1
- 1-	0.5	300	383	400	150	300	0	400	17	400	0	86910
- 2-	0.4	300	383	400	150	333	33	383	0	433	33	91010
- 3-	0.1	300	383	400	150	500	150	300	0	600	0	86280

End of Report

Plant Location with Random Demand

SP_Plant_location.xls [Compatibility Mode] - Microsoft Excel

J7 =WBSP_RAND(1,B14:E14)

Plant Location with Random Demand

Stage 0: We decide which plants to (keep) open, each with a prespecified capacity

Stage 1: Beginning, Demands at various locations are revealed,

Stage 1: End, We satisfy as much demand as we profitably can (by solving a transportation problem).

1) Core Model

	Fxd Cost	Capacity	Open?	Effective Capacity
Atlanta	90	59	0	0
St.Louis	64	65	1	65
Cincinnati	80	65	0	0
Total fixed cost:	64			

Customer Regions

	Chicago	SanAnton	NYC	Miami
Demands	17	17	21	10

Revenues Under Scenario X, per unit shipped:

Atlanta	5	6	5	6
St.Louis	5	8	4	4
Cincinnati	6	7	2	3

Decisions for Scenario X,	Units to ship?				Total Out of	Capacity Constraints
Atlanta	0	0	0	0	0	=<=
St.Louis	17	17	21	10	65	=<=
Cincinnati	0	0	0	0	0	=<=
Total Into:	17	17	21	10	281	**<--Scenario Profit**
Demand UB:	=<=	=<=	=<=	=<=		**(Maximize)**

2) Staging Info

WBSP_VAR	***Declare the stage 0 decisions***
WBSP_RAND	***Declare stage 1 random variables***
WBSP_VAR	***Declare stage 1 decisions***

3) Probability Distn Info

WBSP_DIST_DISCRETE_SV_W	***Declare discrete weighted distribution, jointly***

The random demands, 3 scenarios

Scenario	Chicago	SanAnton	NYC	Miami	Probability
1	12	11	19	16	0.2
2	15	16	22	12	0.5
3	17	17	21	10	0.3

4) Sampling Info

WBSP_STSC

Stage	*Scenario*
1	10

5) Reporting

WBSP_REP	***Reporting cells (optional):***
WBSP_HIST	***Histogram cell (optional):***

WB! Status | WB!_Histogram | WB!_Stochastic | **Model**

Plant Location with Random Demand, Output

The output tab,

WB!_Stochastic, contains two types of information:

1) Various expected values that measure the cost of uncertainty,
2) A scenario by scenario listing of selected variables so we can explicitly verify what happens in each possible scenario.

We may optionally also generate histograms in a WB!_Histogram tab.

Later, we will discuss the various expected values and the various costs of uncertainty.

Plant Location, Scenario Report

SP_Plant_location.xls [Compatibility Mode] - Microsoft Excel

What'sBest!® 10.0.1.2 (Jan 19, 2010) - Library 6.0.1.382 - Stochastic Report -

```
Expected Value (EV)                                      2.666000e+002
Expected Value of Wait-and-See (EVWS)                    2.672000e+002
Expected Value using Expected Value Policy (EVEVP)       2.666000e+002
Expected Value of Perfect Information (=|EVWS-EV|)       6.000000e-001
Expected Value of Modeling Uncertainty (=|EV-EVEVP|)     0.000000e+000
```

REPORTING CELLS

SCENARIO	PROBABILITY	Model!B14 CHICAGO STAGE 1	Model!C14 SANANTON STAGE 1	Model!D14 NYC STAGE 1	Model!E14 MIAMI STAGE 1	Model!B26 STL_CHI STAGE 1	Model!C26 STL_SAN STAGE 1	Model!D26 STL_NYC STAGE 1	Model!E26 STL_MIA STAGE 1	Model!F28 TOTAL_PROFIT STAGE 1
- 1-	0.1	17	17	21	10	17	17	21	10	281
- 2-	0.1	15	16	22	12	15	16	22	12	275
- 3-	0.1	15	16	22	12	15	16	22	12	275
- 4-	0.1	15	16	22	12	15	16	22	12	275
- 5-	0.1	12	11	19	16	12	11	19	16	224
- 6-	0.1	17	17	21	10	17	17	21	10	281
- 7-	0.1	12	11	19	16	12	11	19	16	224
- 8-	0.1	15	16	22	12	15	16	22	12	275
- 9-	0.1	15	16	22	12	15	16	22	12	275
- 10-	0.1	17	17	21	10	17	17	21	10	281

End of Report

WB! Status | WB!_Histogram | WB!_Stochastic | Model

Multi-Stage Portfolio Model with Downside Risk

SP_CollegeDSC.xls [Compatibility Mode] - Microsoft Excel

K10 =WBSP_RAND(1,B10,C10)

Investment Planning for Going to College After 3 Periods.

Two investment options each stage: Stocks and Bonds. Ref: Birge & Louveaux

80	= Goal for wealth at beginning of period 4
4	= Penalty/unit for wealth under goal.
1	= Utility of wealth/unit over goal.

1) Core model

Stage	Growth factor Stocks	Bonds	Beginning Wealth		Total invested	Invest in Stocks	Bonds
0			55	=	55.0000	41.4793	13.5207
1	1.25	1.14	67.26272	=	67.2627	65.0946	2.1681
2	1.25	1.14	83.839905	=	83.8399	83.8399	0.0000
3	1.25	1.14	104.79988				

Under goal:	0		
Over goal:	24.799881	>=	0
Net utility:	24.799881	To be maximized	

2) Time stage specifications for…

Row	Random variables	Decision variables	3) Distribution specifications
9		WBSP_VAR	
10	WBSP_RAND	WBSP_VAR	WBSP_DIST_DISCRETE_SV
12	WBSP_RAND	WBSP_VAR	WBSP_DIST_DISCRETE_SV
14	WBSP_RAND	WBSP_VAR	WBSP_DIST_DISCRETE_SV

Growth factor distribution
(equally likely)

Scenario	Stocks	Bonds
A	1.25	1.14
B	1.06	1.12

4) Sampling specifications by stage

WBSP_STSC

Stage	# Scenarios
1	2
2	2
3	2

5) Reporting Specifications

WBSP_REP

WBSP_HIST

WB! Status | WB!_Histogram | WB!_Stochastic | Model | WB_Hist

Multi-Stage Portfolio Model with Downside Risk

SPCollegeDSC.xls [Compatibility Mode] - Microsoft Excel

A1 | Investment Planning for Going to College After 3 Periods.

Investment Planning for Going to College After 3 Periods.

Two investment options each stage: Stocks and Bonds. Ref: Birge & Louveau

80 = Goal for wealth at beginning of period 4

4 = Penalty/unit for wealth under goal.

1 = Utility of wealth/unit over goal.

1) Core model

	Growth factor		Beginning		Total	Invest in	
Stage	Stocks	Bonds	Wealth		invested	Stocks	Bonds
0			55	=	55.0000	41.4793	13.5207
1	1.06	1.12	59.11124	=	59.1112	36.7432	22.3680
2	1.25	1.14	71.42857	=	71.4286	0.0000	71.4286
3	1.06	1.12	80				

Under goal:	0		
Over goal:	0	=>=	0
Net utility:	0	To be maximized	

2)Time stage specifications for...

Random variables	Decision variables
	WBSP_VAR
WBSP_RAND	WBSP_VAR
WBSP_RAND	WBSP_VAR
WBSP_RAND	WBSP_VAR

3) Distribution specifications

WBSP_DIST_DISCRETE

WBSP_DIST_DISCRETE

WBSP_DIST_DISCRETE

Growth factor distribution (equally likely)

Scenario	Stocks	Bonds
A	1.25	1.14
B	1.06	1.12

4) Sampling specifications by stage

WBSP_STSC

Stage	# Scenarios
1	2
2	2
3	2

5) Reporting Specifications

WBSP_REP

Stage Assumptions

1) Default stage assignment for a formula or constraint = highest stage of any variable in the constraint or RHS of the formula, except,

2) Objective function is automatically assigned to stage 0.

WBI Status | WBI Stochastic | Core model | Report

Multi-stage Portfolio: Solution and Policy

SP_CollegeDSC.xls [Compatibility Mode] - Microsoft Excel

H19 Model!D12

```
RANDOMS                        6
STAGES                         4
NODES                         15
SCENARIOS                      8

EXPECTED VALUE                                    -1.514085e+000
Expected Value of Perfect Information Lower Bound  1.201109e+001

     REPORTING CELLS
SCENARIO
```

		Model!F9	Model!G9	Model!H9	Model!D10	Model!G10	Model!H10	**Model!D12**	**Model!G12**	**Model!H12**	Model!D14	Model!D16	Model!D1
		WEALTH0	STOCKINVEST0	BONDINVEST0	WEALTH1	STOCKINVEST1	BONDINVEST1	**WEALTH2**	**STOCKINVEST2**	**BONDINVEST2**	WEALTH3	UNDERGOAL	OVERGOA
		STAGE 0	STAGE 0	STAGE 0	STAGE 1	STAGE 1	STAGE 1	**STAGE 2**	**STAGE 2**	**STAGE 2**	STAGE 3	STAGE 3	STAGE
-	1-	55	41.479272	13.520728	59.111244	36.743215	22.368029	**71.428571**	**0**	**71.428571**	80	0	
-	2-	55	41.479272	13.520728	59.111244	36.743215	22.368029	**71.428571**	**0**	**71.428571**	81.428571	0	1.42857
-	3-	55	41.479272	13.520728	59.111244	36.743215	22.368029	**64**	**64**	**0**	67.84	12.16	
-	4-	55	41.479272	13.520728	59.111244	36.743215	22.368029	**64**	**64**	**0**	80	0	
-	5-	55	41.479272	13.520728	67.26272	65.094582	2.168138	**83.839905**	**83.839905**	**0**	88.870299	0	8.87029
-	6-	55	41.479272	13.520728	67.26272	65.094582	2.168138	**83.839905**	**83.839905**	**0**	104.799881	0	24.79988
-	7-	55	41.479272	13.520728	67.26272	65.094582	2.168138	**71.428571**	**0**	**71.428571**	80	0	
-	8-	55	41.479272	13.520728	67.26272	65.094582	2.168138	**71.428571**	**0**	**71.428571**	81.428571	0	1.42857

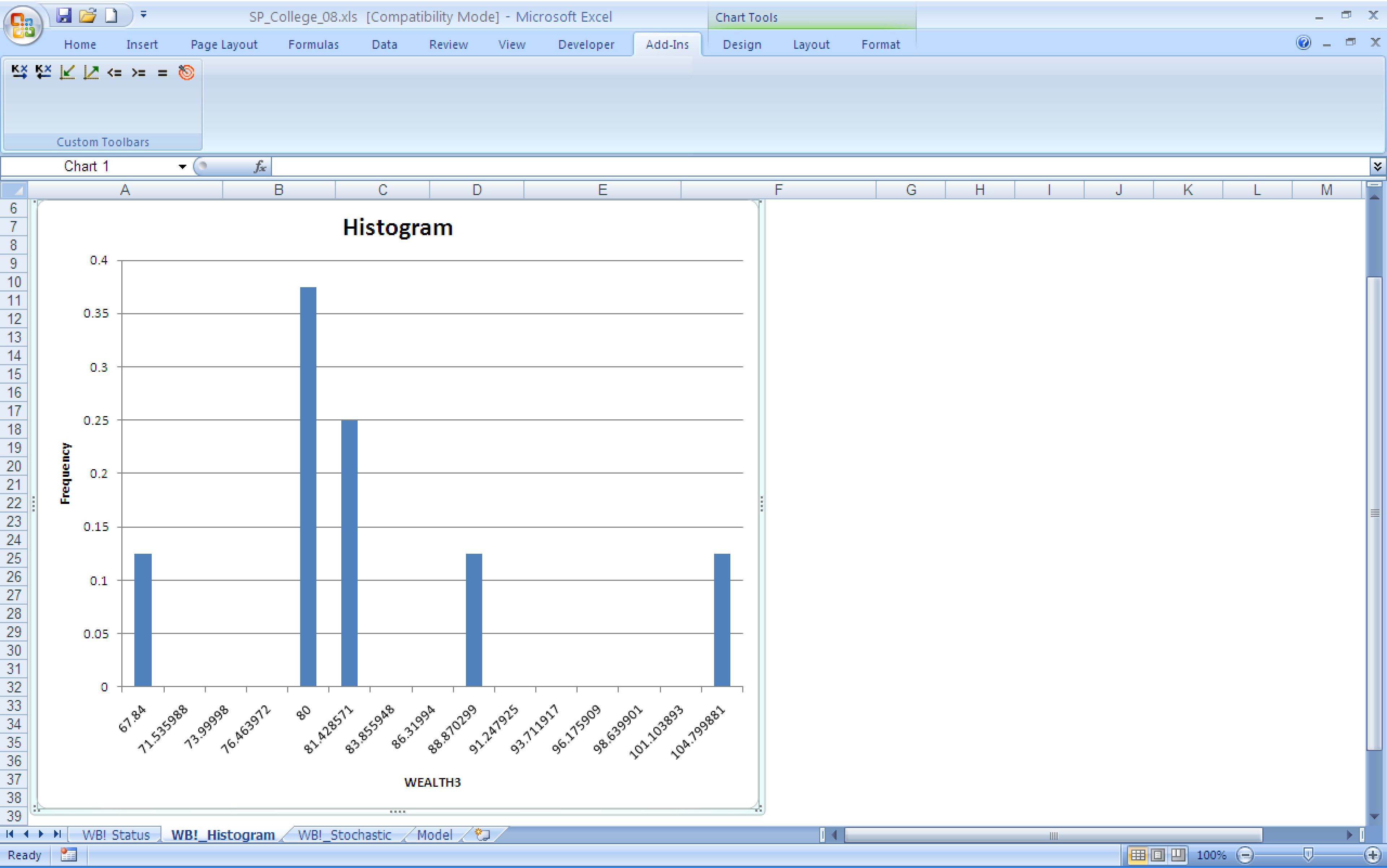
End of Report

Notice when we put all our money in stocks in stage 2….

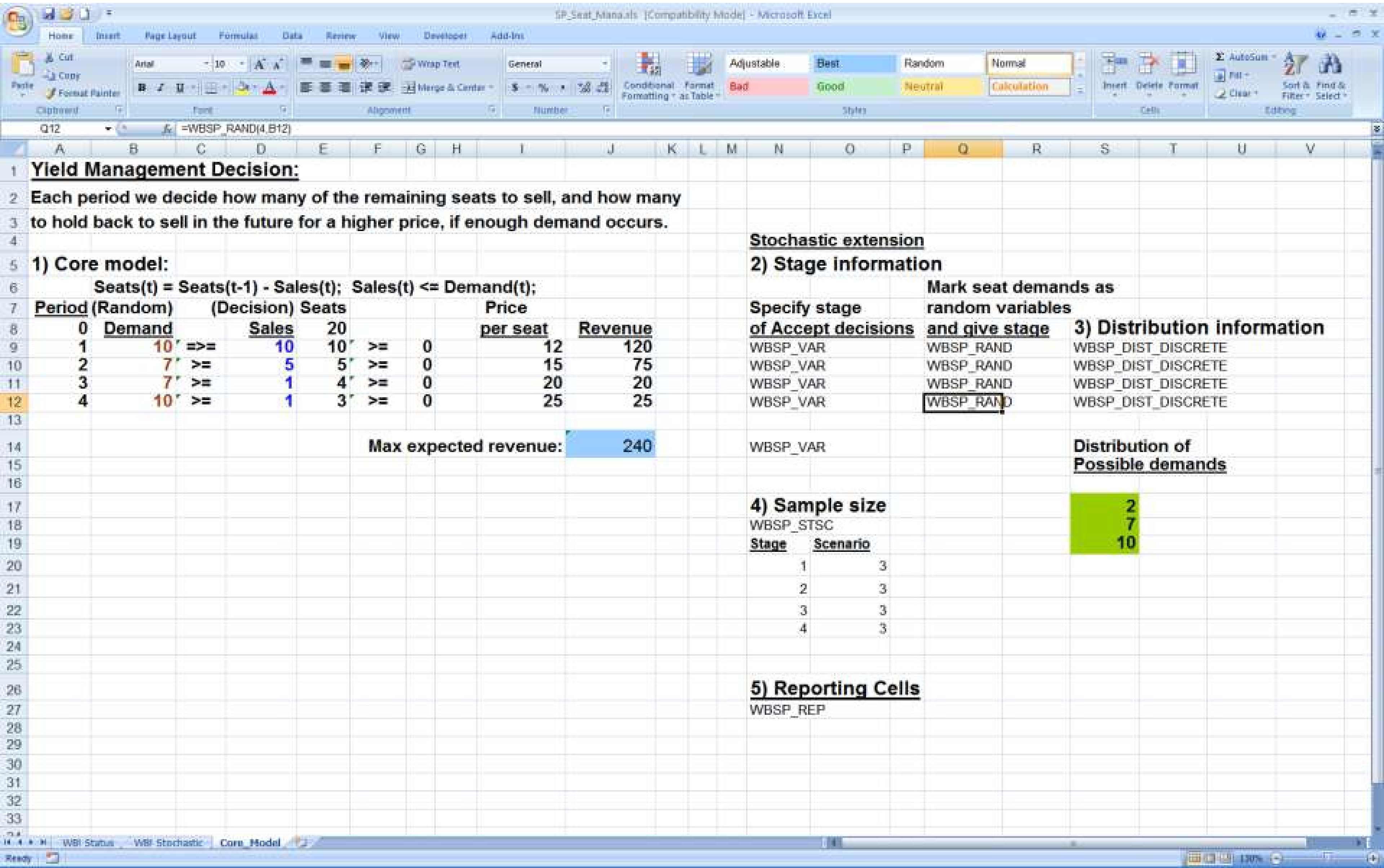
WB! Status | WB!_Stochastic | Model | WB_Hist

Ready — Average: 48.44950783 Count: 33 Sum: 1162.788188 120%

Terminal Wealth Distribution: College/Retirement Planning



Yield Management: Bird in Hand vs. Future Bird in Bush



Q12 =WBSP_RAND(4,B12)

Yield Management Decision:

Each period we decide how many of the remaining seats to sell, and how many to hold back to sell in the future for a higher price, if enough demand occurs.

1) Core model:

Seats(t) = Seats(t-1) - Sales(t); Sales(t) <= Demand(t);

Period	(Random) Demand		(Decision) Sales	Seats			Price per seat	Revenue
0				20				
1	10	=>=	10	10	>=	0	12	120
2	7	>=	5	5	>=	0	15	75
3	7	>=	1	4	>=	0	20	20
4	10	>=	1	3	>=	0	25	25

Max expected revenue: 240

Stochastic extension

2) Stage information

Specify stage of Accept decisions	Mark seat demands as random variables and give stage	3) Distribution information
WBSP_VAR	WBSP_RAND	WBSP_DIST_DISCRETE
WBSP_VAR	WBSP_RAND	WBSP_DIST_DISCRETE
WBSP_VAR	WBSP_RAND	WBSP_DIST_DISCRETE
WBSP_VAR	WBSP_RAND	WBSP_DIST_DISCRETE

WBSP_VAR

Distribution of Possible demands

2
7
10

4) Sample size

WBSP_STSC

Stage	Scenario
1	3
2	3
3	3
4	3

5) Reporting Cells

WBSP_REP

Markowitz Portfolio with Min Buy/Cardinality Constraints

Minimize $x^T Q x = \sum_i \sum_j q_{ij} * x_i * x_j$,

$\sum_i x_i = 1$, ! Budget constraint, Stage 0;

$\sum_i \mu_i * x_i \geq \rho$, ! Expected return of portfolio, Stage 1;

! Complicating constraints:

$L_i y_i \leq x_i \leq U_i y_i$, $i = 1, \ldots, n$! If buy any, must buy at least L_i ;

$y_i = 0$ or 1, $i = 1, \ldots, n$

$\sum_i y_i \leq K$, ! Cardinality constraint;

Q is a Positive Semi-Definite n by n matrix of the covariances of n assets.
μ_i = expected return of asset i during the investment period,
ρ = target expected return,
L_i = minimum bought of asset i, if any of it is bought,
U_i = maximum quantity e.g., 1, that can be bought of asset i.
K = upper limit on number assets in portfolio.

This would be an easy convex quadratic problem if it were not for the complicating constraints. LINDO API 9 has much improved methods for finding good solutions quickly to problems of the above type.

Markowitz Portfolio with Min Buy/Cardinality Constraints-II

Below are some results from letting LINDO API 8 and LINDO API 9 run for at most 300 seconds on a set of problems of the above type. Each problem had from 20 to 400 assets, as indicated in the Problem name.

Best Results in 300 seconds.

Problem	Best known solution	API 8/LINGO14 Best soln	API 9/LINGO15 Bound	API 9/LINGO15 Best soln	API 9 Time (sec) to best
Portdiagcard20	0.11022	0.11022	0.11022	0.11022	2
orl200-005-a	66.94	93.76	66.53	66.94	25
orl200-05-c	49.14	54.34	47.14	58.85	10
orl200-05-f	53.35	55.66	50.97	53.88	151
orl300_005_b	98.71	126.06	96.13	98.71	47
orl300_05_e	76.26	94.88	74.32	76.43	158
orl400_05_d	98.89	110.53	96.93	99.49	147
pard200_a	186.00	298.88	184.51	186.00	33
pard300_h	268.86	327.02	265.63	269.85	50
pard400_d	356.90	610.67	355.18	356.90	121
pard400_j	357.45	529.31	352.94	357.44	152

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