Application Survey Paper

Applications of Optimization to Problems in Sales, Pricing, and Marketing

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Some of the earliest applications of optimization have been in marketing, starting with advertising media selection models in the 1970's. In the first section we give brief descriptions of some of these applications, as well as pointers on how to learn more. In the second section we give simple examples.

Section I

1) Advertising media selection: If we are given: a) set of market segments we wish to reach, and b) a set of advertising media and some measure of how many people each medium reaches in each market segment, and c) an advertising budget, then we want to decide how much to spend in each medium so that we maximize the value of the people that we reach, subject to not exceeding our advertising budget. In practice, a number of additional details are taken into account such as the decreasing returns to additional exposures in a given market. See the text "Optimization Modeling with LINGO" for an example.

2) Sales territory design/realignment: If we are given a list of (potential) customers, an estimate of how much work or time needs to be allocated to each customer per month, an estimate of the value of each customer, then allocate customers to sales representatives(reps) so that a) the work load is approximately equal among reps, b) the total value assigned to each rep is approximately equal, and b) the region assigned to each rep makes geographic sense, e.g., the customers are close together or contiguous. This approach has been used with some success in the pharmaceutical industry. Some of the ideas here also apply to political redistricting, where important considerations are: 1) Size Similarity: the population assigned to each cluster should be close to the same for all clusters, 2) Closeness: the average distance of the units in a cluster from the cluster center should be small, e.g., if transport costs are important, 3) Connected: Each cluster should be connected geographically, i.e., for any two units in the same cluster, there must be a path connecting them, 4) Compactness: The total boundary length of the clusters should be small. 5) Feature Similarity: The maximum width of cluster by some measure, e.g., distance or time zone difference, should be small, 6) Gerrymandering feature: If each unit has some fraction of its population belonging to our political party, we want to have a large number of clusters in which our party has the majority.

3) Sales call planning: This is the detailed, tactical, or daily version of the sales territory problem. If we are given a small set of customers who are good candidates for a given sales rep to visit today, then which ones should the rep visit and in what sequence so that the work fits into the time available for the day and we maximize the value of the customers visited minus the travel costs of visiting the customers.

4) Revenue Management: This methodology got its start in the airline industry and has expanded to a variety of other industries, particularly to products that "expire", such as seats on a particular flight, rooms in a hotel on a particular night, tickets to an event on a particular day, etc. In its simplest form RM takes into account the fact that there are some customers who will be willing to pay a very high price for your commodity just a few days or minutes before the flight departs or sports event. Unfortunately you have more tickets to sell than there are people who are willing to pay the high price, so some of the tickets will have to be sold at a lower price. The optimization problem in its simplest form is deciding how many tickets to offer at the low price and how many tickets to hold back to hopefully sell at the higher price with the expectation that enough people will show up "just before show time" to buy the high priced tickets you have held back for them. See "Optimization Modeling with LINGO" for examples, as well as simple example in the second section.

5) Overbooking limits: This may be considered as a special but older case of revenue management. Again, first used in the airline industry, it addresses the situation where you sell in advance a limited number of some commodity. If a nontrivial number of advance purchasers do not show up to claim their purchase, then you have the overbooking optimization problem of deciding how many units to sell beyond the actual number available, balancing the opportunity cost of letting a unit go unused vs. the very real cost of dealing with a customer who shows up and finds unavailable the product that she expected. See "Optimization Modeling with LINGO" for detailed examples.

6) Multi-product and Bundle Pricing: When a firm is selling multiple products, the firm should take into account how some products are complements and some are substitutes. Bundle pricing is a form of quantity discounting. Under bundle pricing, a buyer can buy a combination of products, e.g., a word processing program and a spreadsheet program for less than the sum of prices of these products bought individually. If customers have diverse needs, e.g., some customers are willing to pay a lot for a word processing program and only a modest amount for a spreadsheet, whereas other customers are willing to pay a lot for a spreadsheet but only a modest amount for the word processor, then bundle processing may help. The optimization problem for the seller is to set prices for the individual products and for the various bundles so as to maximize total revenues, taking into account market segment sizes and that each market segment will buy that product or bundle that gives it the best deal in terms of value minus cost. It is not unusual for a vendor to be able to increase total revenues by almost 50%, i.e., by a factor of almost 1.5, by using bundle pricing. See the text "Optimization Modeling with LINGO" for some detailed examples.

7) Forecasting of new product acceptance: When a new product is introduced there is a question of forecasting the sales each week. Bass introduced a popular model to predict the S shaped total acceptance curve that one typically sees for a new product. This model was originally used in predicting the acceptance of new styles of home appliances, as well as color televisions when they were first introduced. The basic idea is to describe the market by three numbers: M = the potential/final market size, p = the fraction of the market who are innovators and are likely to try a new product in any given period just because the product is available or heavily advertised, and q = a measure of the "word of mouth" effect or the fraction of the market who are imitators and will try the product only because someone else told them the product was good. A nonlinear optimization problem arises if one has preliminary sales data, or historical data on a similar product and one wishes to choose values for M, p, and q so as to minimize the retrospective fitting error, and also hopefully produce a good forecast of sales next month. See the Bass Model in the Applications Library at http://www.lindo.com.

8) Credit scoring and classification: In sales, one may wish to classify potential customers into two or more classes, e.g., credit worthy or not, based on various attributes of the customer, e.g., educational level, salary, etc. Finding an appropriate "scoring formula" can be attacked in various ways, with Logit and Probit analysis being perhaps the most well known. As an optimization problem in its simplest form it is: given a set of historical data including attributes of customers and their actual classification, choose a weight for each attribute and a score threshold so that some measure of misclassification cost is minimized. See the models Logit.lng and Probit.lng in the Applications Library at http://www.lindo.com

9) Conjoint analysis: This methodology is concerned with how to position, and to some extent design, a product that has lots of attributes so that it comes close to matching the ideal product desired by the customer. A well-known application of these ideas was to the design of the "Courtyard by Marriot" chain of business hotels. Potential customers have difficulty giving a complete specification of what is important to them in a product, so conjoint analysis asks one or more customers to make one of more simple comparisons between slightly different products, e.g., differing length of warranty, different price, different weight or size, etc. Given these ranking results, we want to choose a set of weights for the features so as to minimize the extent to which this weighting violates the rankings given by the customers. For examples, see the models conjoints.lng and conjointg.lng in the Applications Library at http://www.lindo.com

10) Product mix problems: This is the problem of deciding how much of which products to produce or sell in a product line. If different products use different amounts of different scarce production capacities, then maximize profit subject to these production constraints is a linear program. U.S. automobile manufacturers for example have an interest complication in this regard in that there are Federal limits on average fuel economy taken over all vehicles the manufacturer sells in the U.S. So for example, a manufacturer may wish to sell a number of moderately profitable high fuel economy cars because this will allow it to sell high profit, low fuel economy cars. See “Optimization Modeling with LINGO” for detailed examples.

11) Shelf space allocation: The manager of a retail store such as a supermarket must decide both where to place each stock keeping unit (SKU) on the shelf but also how much space or “facings” to allocate to the SKU. Products that are subject to impulse purchases should be placed in high traffic aisles and/or at eye level. SKU’s that appeal to children may be put on lower shelves. SKU’s that have high sales volume should receive more shelf facings because that will tend to reduce the labor devoted to keeping the SKU in stock. If you can quantify the value of giving each SKU a certain number of facings at each possible shelf location, then you can maximize the value of the product shelf positioning, subject to not exceeding the available shelf space.

12) Auctions: There are certain situations when an auction is a good way of “clearing” a market. When one is selling multiple objects and there are various substitution and complementarity interactions among the objects being sold, then linear and integer programming are good methods for deciding who gets what and at what price. See for example the paper on multi-object auctions, as well as the model, aucteq.ltx, in the Applications Library at http://www.lindo.com.

13) Survey Sample Balancing: Suppose you take a random survey sample of public opinion of purchase probability of a proposed new product. After you collect the results, you find that of your 100 observations, 55 are male and 45 are female. You know that the actual population has 0.50 fraction male and 0.50 fraction female. You might reasonably decide to give equal weight to the male portion of the sample and the female portion of the sample, i.e., we would "balance" the sample so that it matches the population. If we did no balancing we would give weight 1.0 to every observation. For our little example, balancing would give weight 100*0.5/55 = .90909 to each of the 55 male observations, and weight 100*0.5/45 1.11111 to each of the 45 female observations.

In the general case, rather than one characteristic, e.g., gender, we have I different characteristics (also called variables), and rather than two levels, e.g. male or female, we have *m* possible levels (also called brackets). For example, we might have 3 characteristics or variables: age bracket, region, and income bracket. For each characteristic we might allow 5 different levels or brackets. Given the fraction of the population that is in each bracket for each characteristic, and a sample, and knowing the age level, region, and income bracket for each observation in the sample, how much weight should be given to each observation, so as to come close to matching the population fractions for each bracket?

14) Mark Down Pricing: A seller wants to set prices for a single product in two periods, just before Christmas and just after Christmas. We consider the full knowledge case. The potential buyers know there will be markdowns in the second period. For simplicity, we assume the potential buyers will know what the prices will be in the second period when they are deciding to buy or not in the first period. Each buyer will buy the product, if she buys any, in the period which gives the greater consumer surplus, i.e. reservation price - purchase price. Typically, a buyer will be willing to pay more, perhaps substantially, in the first period. E.g., the buyer is willing to pay more for a ski parka at the beginning of ski season, than at the end. If you are an economist, you can also think of this as a Stackelberg game in which the seller is the leader who makes the first decisions: the prices in the two periods, and then the buyers optimize for themselves, given the prices.

Section II

Revenue Management Example: Basic problem: Given

a) set of limited resources, e.g., seats on airplane flights, hotel rooms, rental cars over several days, and
b) different types of customers needing different combinations of our resources, and
c) each customer type having an amount they are willing to pay for their needed combination,

what volumes should we sell of each product bundle to each customer class?

In practice, the dual or shadow prices on the limited resources in the solution may be as useful as the allocations to customers. The prices can be used to "price out" a new customer request to decide if it is attractive relative to other opportunities to sell or rent our resources.

Specific example: Consider an airline having a set of three flights, A to C, B to C, and C to D to serve four cities, A, B, C, and D, in the network:

```
A
 \
  C - D
 /
B
```

The airline uses a two-fare/class pricing structure. The decision of how many seats or capacity to allocate to each price class is sometimes called yield or revenue management. The respective flight capacities are 100, 110, and 120. How many seats should be allocated to each class on each of the three flights to each of the 5 itineraries?

In the data below, for a given itinerary, a first class customer always gives more revenue than a second class customer. Thus, a plausible rule would be to first satisfy first class as much as possible, and then fill any remaining seats with second class.
Will this lead to the most profitable solution?

The data are:

```
 DATA:
    LEG =    AC    BC    CD; ! The legs in network, or objects to sell;
   TCAP =   100   110   120; ! Capacity or availability of each leg/object;
  CLASS = ONE TWO;           ! The classes;
ITNRY = IAC  IBC  IAD  IBD  ICD; ! The itineraries or bundles of objects;
   RP  = 140  186  244  272  165  ! and the reservation  ;
          80  103  193  199   90; !  prices of 2 classes;
   DEM =  11   16   24   12   47  ! Max # people willing to buy an;
          49   17   53   67   56; ! itinerary/bundle at that price.

 ! The  (Itinerary, leg) combinations;
    IXL= IAC, AC
         IBC, BC
         IAD, AC IAD, CD
         IBD, BC IBD, CD
         ICD, CD ;
ENDDATA
```

The solution is:

Class	Bundle	Units_to_Sell	Revenue
ONE	IAC	11	1540.00
ONE	IBC	16	2976.00
ONE	IAD	24	5856.00
ONE	IBD	12	3264.00
ONE	ICD	3	495.00
TWO	IAC	49	3920.00
TWO	IBC	17	1751.00
TWO	IAD	16	3088.00
TWO	IBD	65	12935.00
TWO	ICD	0	0.00
Total expected revenue=			35825.00

Resource	ShadowPrice
AC	28
BC	34
CD	165

Multi-product and Bundle Pricing Example: Basic problem:

A producer wants to choose a price for each of a number of products, based on knowledge of the "willingness to pay" (Reservation price) of various market segments for each of the products. The producer wants to choose a price for each product/bundle so as to maximize total revenue minus costs. You can think of a product as a bundle of features.

Each customer segment behaves as follows:

If the price of a product/bundle is > than a customer's reservation price, then the customer will not buy that product.

Each customer, *i*, will buy the one product/bundle, *j*, that maximizes his consumer surplus (= reservation_price(*i*,*j*) minus price *i* must pay for *j*.)

Each customer/market segment is described by:

Its size,

Reservation price for each product;

Here is an example data set:

```
DATA:
   CUST = HOME     STUD     LEGL     BUSN; ! Customer/market segments;
   SIZE = 5900     3000     2000     3100; ! Customer/market segment sizes;

! We sell three basic products: Spreadsheet, Wordprocessor, Presenter software,
   as well as all combinations of the three features;
  BUNDLE =    BS     BW     BP    BSW    BSP    BWP   BSWP; ! Products/bundles;
! Reservation/max_willing_to_pay prices of markets for products;
     RP =    190     90     50    280    240    140    330  ! Home;
             160    180     70    340    230    250    410  ! Student;
              50    250     85    300    135    335    385  ! Legal;
             300    100    120    400    420    220    520; ! General Business;
! Variable costs of each product;
   COST =    100     20     30    120    130     50    150;
ENDDATA
```

The profit maximizing solution is:

```
 Max profit for vendor= 2729000

 Profit maximizing price for each bundle:
          BS     BW     BP    BSW    BSP    BWP   BSWP
         190    230     65    280    300    315    400

 Purchase matrix:
          BS     BW     BP    BSW    BSP    BWP   BSWP
  HOME                          1
  STUD                          1
  LEGL                                        1
  BUSN                                               1


 Consumer surplus, Market segment vs. Product/Bundle:
          BS     BW     BP    BSW    BSP    BWP   BSWP
  HOME     0   -140    -15      0    -60   -175    -70
  STUD   -30    -50      5     60    -70    -65     10
  LEGL  -140     20     20     20   -165     20    -15
  BUSN   110   -130     55    120    120    -95    120
```

Mark Down Pricing Example: Basic problem:

A seller wants to set prices for a single product in two periods, e.g., just before Christmas and just after Christmas. We consider the full knowledge case. The potential buyers know there will be markdowns in the second period. For simplicity, we assume the potential buyers will know what the prices will be in the second period when they are deciding to buy or not in the first period. Each buyer will buy the product, if she buys any, in the period which gives the greater consumer surplus, i.e. reservation price - purchase price. Typically, a buyer will be willing to pay more, perhaps substantially, in the first period. E.g., the buyer is willing to pay more for a ski parka at the beginning of ski season. If you are an economist, you can think of this as a Stackelberg game in which the seller is the leader who makes the first decisions: the prices in the two periods, and then the buyers optimize for themselves, given the prices.

Here is an example data set:

```
DATA:
! Reservation prices for the customer groups in each period;
 CUSTG= CG1  CG2  CG3  CG4;
 R1 =   220  210  180  175; ! Period 1;
 R2 =   150  160  175  150; ! Period 2;
! Number of people in each group;
 NP =    50   90  110   35;
! Cost per unit to the vendor;
 COST = 60;
ENDDATA
```

The profit maximizing solution is:

```
Two Period Pricing Problem.
 Reservation price
  by group.
           Period 1   Period 2
   CG1      220.00     150.00
   CG2      210.00     160.00
   CG3      180.00     175.00
   CG4      175.00     150.00
  Cost/unit=  60.00

 Results:
   Price:     210.00     175.00
 Units purchased:
   CG1           50          0
   CG2           90          0
   CG3            0        110
   CG4            0          0

 Total revenue= 48650.00
 Total cost   = 15000.00
 Profit       = 33650.00
```