Optimization Applications in Finance

Linus Schrage

linus.schrage@gsb.uchicago.edu

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Intended Audience

Practitioners seeking broader range of applications;

Instructors seeking wide range of examples for class;

Researchers seeking some non-well solved problems.

Talk/paper is available in full at:

http://www.lindo.com/finapm1.pdf

Paper contains extensive list of references.

Models, in modeling language form and spreadsheet form for most applications discussed, are at:

http://www.lindo.com/library.html.

Topics

Financial portfolios
- Covariance/Markowitz based
  - Scenarios version,
  - Factors version,
  - transaction costs and taxes,
  - Max number of assets/cardinality
  - Tracking, matching, and hedging.
- Value at Risk portfolios

Project Selection/Budgeting

Portfolio Repackaging/Pooling

Cash Flow Matching Bond Portfolio

Muni-Bond package bidding
- Net interest cost
- True interest cost

Optimal Lease Structuring

Flexible spending

Sealed Bid Auctions
Where value of an object
depends upon other objects obtained by bidder.

Market Equilibria and Pricing
Finite alternatives
Continuous alternatives

Options/Derivatives
- Stock prices
  - European option
  - American option
  - Bermuda option
  - Asian option
    - Quasi-random numbers
  - Weather options/insurance
- Interest rates
- Foreign exchange rate

Building scalable models in spreadsheets

Financial Portfolios - Overview

Basic Covariance/Markowitz based.
  Minimize variance;
   Subject to:
  Amount invested = 1;
  Expected return   target;

 Problems with it:
  How do you estimate the covariance matrix?
   e.g., for internet stocks.

  How often to rebalance,
   transaction costs, taxes.

  Is variance the best measure of risk?

 Some approaches:
  Scenarios version,
  Factors version,
  Transaction costs and taxes,
  Max number of assets/cardinality
  Tracking, matching, and hedging.

`Models:  portcovr`

Scenario Approach - Overview

Construct a number of scenarios of possible outcomes.
Each scenario has a probability.
Optimize using this discrete distribution.
Based on idea of stochastic programming.

A bit more mathematically:

$$
\begin{aligned}
& Min = \sum_s p_s D_s^2 \\
& s.t. \\
& \sum_j x_j = 1, \\
& \sum_s p_s R_s \geq t, \\
& R_s = \sum_j r_{js} x_j, \\
& \overline{R} = \sum_s p_s R_s, \\
& D_s = R_s - \overline{R};
\end{aligned}
$$

Advantages:

Easier to use if subjective data must be used.
If little historical data, it is difficult to derive a sensible covariance matrix, e.g., a subjective one may be not positive definite.

Richer class of objectives are possible, e.g., arbitrary utility functions, downside risk, semi-variance,
LP rather than QP may be used.

Some results:

If original data, used in computing covariance matrix, are used as scenarios, then scenario approach recommends exactly the same portfolio as the Markowitz QP approach.

Given a covariance matrix and a return vector, a set of scenarios can be constructed that exactly matches the covariance matrix.

Ref.: Markowitz and Perold (1981), Carino, et. al.(1994), Schrage(1999).

`Model: prtscen`

Portfolios with
Transaction Costs and Taxes - Overview

Our real concern is growth after
paying taxes and transaction costs.

Some mutual funds track the S&P 500 while generating very little dividend income and no <u>realized</u> capital gains, so almost no tax.

The model:

Minimize risk;

Sales   Purchases + transaction costs;

Expected return - taxes   target;

Taxes   (tax rate) * (dividend + capital gain income);

Questions and comments:

How often to rebalance is answered.

Churning a portfolio may be useful
for an individual portfolio.

Bid/Ask spread handled similar to explicit transaction cost.

Taxes introduce a nonconvexity, e.g.,
sell a stock to get a tax loss and then buy it back.

`Model:  portax`

Portfolios & Cardinality Constraints - Overview

It may be expensive to have an asset in a portfolio,
- monitoring costs,
- proxy for transaction costs.
- Warren Buffett had 1/4 of his portfolio in Coke.

Popular approach:

Portfolio is restricted to at most 20, say, assets,

i.e., the cardinality constraint:

$$\sum_j \delta(x_j) \leq 20;$$

Example: 19 stock portfolio with
no cardinality constraint, s.d. = .4952,
stocks $\leq$ 7, s.d. = .4960,
stocks $\leq$ 4, s.d. = .5058.

It is easy to get good solutions, but difficult computationally to get optimal solutions.

Ref., Bienstock(1996), Bertsimas, et. al.(1999)

`Model: portcard`

Matching and Hedging Portfolios - Overview

Matching: A portfolio manager wants to obtain results
no worse than some target porfolio.

Hedging: You want your portfolio to be
negatively correlated with some target portfolio.

Some results:

If the target is not on the risk/return efficient frontier, then
The matching portfolio is probably not on the efficient frontier.

If the risk free asset is available, then finding the portfolio on the efficient frontier that is closest to the target portfolio is easily computed.

Ref.: Schrage(1999)

`Model: porthedg, portmtch.`

Value at Risk - Overview

Popularized by J.P. Morgan
(http://www.jpmorgan.com) in 1994 with
RiskMetrics™ system.

Value at Risk"(VaR) requires two numbers:
1) an interval of time, e.g., one day,
over which you are concerned about losing money, and
2) a probability threshold, e.g., 5%,
above which you care about harmful outcomes.

VaR is then defined as that amount of loss in one day that has probability at most 5% of being exceeded.

Questions:
When is it equivalent to traditional methods,
e.g., Markowitz?

When is it dangerous?

Some results:

Minimize VaR is equivalent to
Minimize $Z_{\mapsto}$ * s.d. - expected return,

So it produces a portfolio on the efficient frontier.

But may cause you to choose a fat tailed distribution, e.g.,
choose a distribution with Prob{Loss = $100M} = .049,
over one for which Prob{Loss = $1M} = .051.

Ref: Jorion(1997)

`Model:  portvar`

Project Selection Portfolios - Overview

Many firms, e.g. local phone company, use a

Yearly budgeting process to

Select projects to fund for the year.

Each project has a:
Cost,
Rate of return
Risk
Interaction with other projects.

Questions:
How to efficiently and intuitively
represent the interactions,
represent uncertainty?
Choice of criterion/risk measure.

Not so good idea:
Min s.d./return
(Reallocate with some money in T-bills might yield
same return but lower s.d.)

`Model: capbudx`

Cash Flow Matching Bond Portfolio - Overview

Situation:

Faced with a stream of future cash flow requirements.

Want to set aside a minimum lump sum now to guarantee ability to match these requirements.

Ex.: Injury law suits, fund to payoff lottery winners,
trust fund for college, remove debt from balance sheet.

Approaches:

a) Compute present value of stream.
What should be the interest rate? Passbook savings?

b) Buy a portfolio of high quality bonds so cash throw-offs match the needs.
This is an LP, or easy IP.

General form:
Minimize Initial investment
s.t.
For each period:
Interest and principal from bonds + short term money returns
external cash needs + short term investments.

Issues: Choice of period length, treatment of taxes.

`Model: pbond.`

Optimal Lease Structuring - Overview

Firm A(Lessee/renter): Needs equipment,
does not need tax benefits, e.g., depreciation.

Firm B(Lessor/owner): Can use the tax benefits of ownership,
e.g., is making a profit overall.

How should stream of payments from Lessee to Lessor be structured so as to:

Minimize PV of payments by Lessee,
Maximize PV of after-tax benefits to Lessor,
Satisfy IRS definition of a lease.

General form of IRS constraints:
yearly payments cannot fluctuate wildly.

Ref. Litty(1994).

Muni-Bond Package Bidding - Overview

City specifies:
How much money it wants now, face value of bonds,
Maturities at which it will pay back principals,
Restrictions on interest rates.

Broker/bidder proposes interest rate for each maturity, so as to
Make a target profit when reselling to market,
Make attractive to city.

Cities historically had used
Net interest cost to measure attractiveness
True interest cost/IRR is now common.

Bidder formulation of problem:

Maximize attractiveness to city
s.t.
Make target profit,
Satisfy interest rate constraints.

Ref: Nauss(1988), http://www.muniauction.com

`Models: munbndn, munbndt`

Sealed Bid Auctions - Overview:

Situation:

Several objects to be sold, e.g., nearby real estate parcels, radio frequency spectrum in a metro area.

Value of an object to bidder depends upon which other objects the bidder wins.

Solution methods

open auction style: First sell all objects as individuals, then sell all as a bundle, Pick best.

Optimization with sealed bids:

Simple bids,

k out *n* bids: bidder says

"I want exactly *k* out of these *n* objects",

where *k* is usually either 1 or *n*.

Issues:

Which dual prices should you use as market prices?

$$\begin{array}{llll} \text{Max} & 200\,B1 + 200\,B2 + 100\,B3 & & \\ \text{St} & & & \\ & B1 \qquad\qquad + B3 \le 1 & & \\ & \qquad B2 \quad + B3 \le 1 & & \\ & 0 \le B1, B2, B3 \le 1 & & \end{array}$$

Possible prices

I	II	III
\$200	\$100	\$50
\$200	0	\$50

Resolution: Use generalized Vickrey prices?
Use a floating capacity perturbation.

Options/Derivatives - Overview

A bet on an outcome
derived from an underlying,
fundamental financial random variable.

Example:
Option to exercise in some future interval the right to
buy a stock at a guaranteed price,
sell a commodity at a guaranteed price.

How can optimization-like methods help?
Note the basic steps are:
1) Create a generic model of the fundamental r.v.,
(Once a career.)
2) Fit the model to today's data.
(Once a day.)
3) Price a derivative instrument based on today's fit.
(Several times/day.)

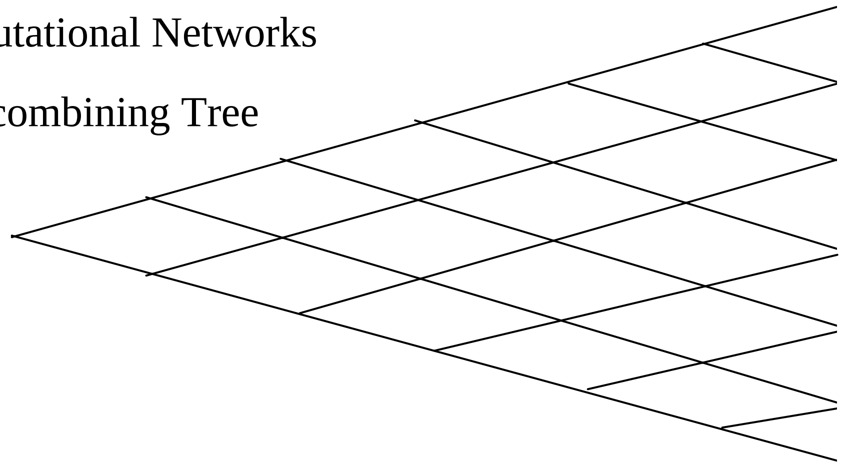
Optimization-like methods can help with (2) and (3).

Why numerical methods, e.g. binomial "tree" rather than
closed form solutions such as Black/Scholes formula?

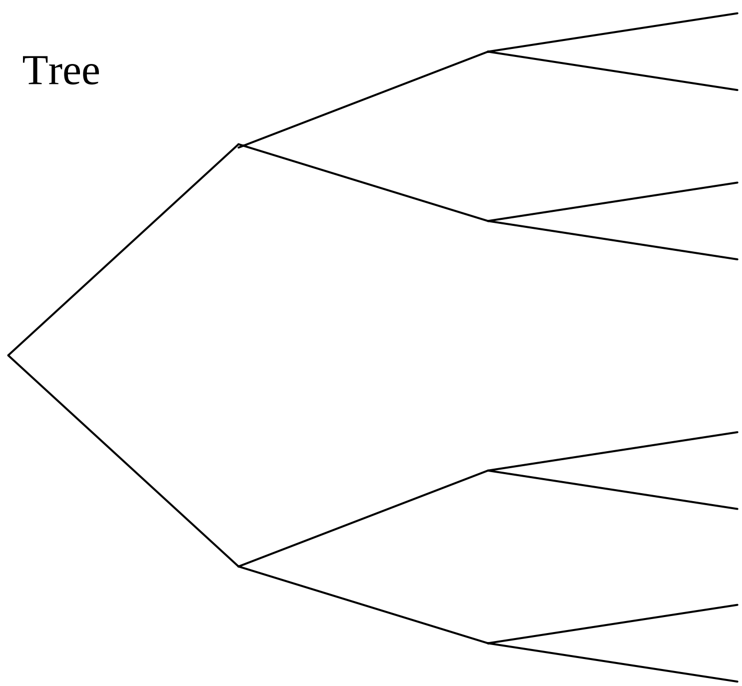
Can evaluate a greater variety of instruments.
B/S formula is quite good for European options,
almost useless for more exotic, e.g., Asian options.

Computational Networks

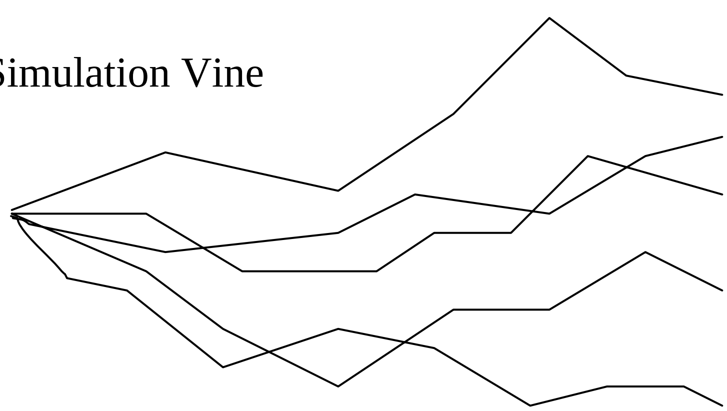
Recombining Tree



Tree



Simulation Vine



When using simulation, should consider using
Quasi-random numbers, Ref: Press et. al.(1992).

Most common underlying stochastic processes:

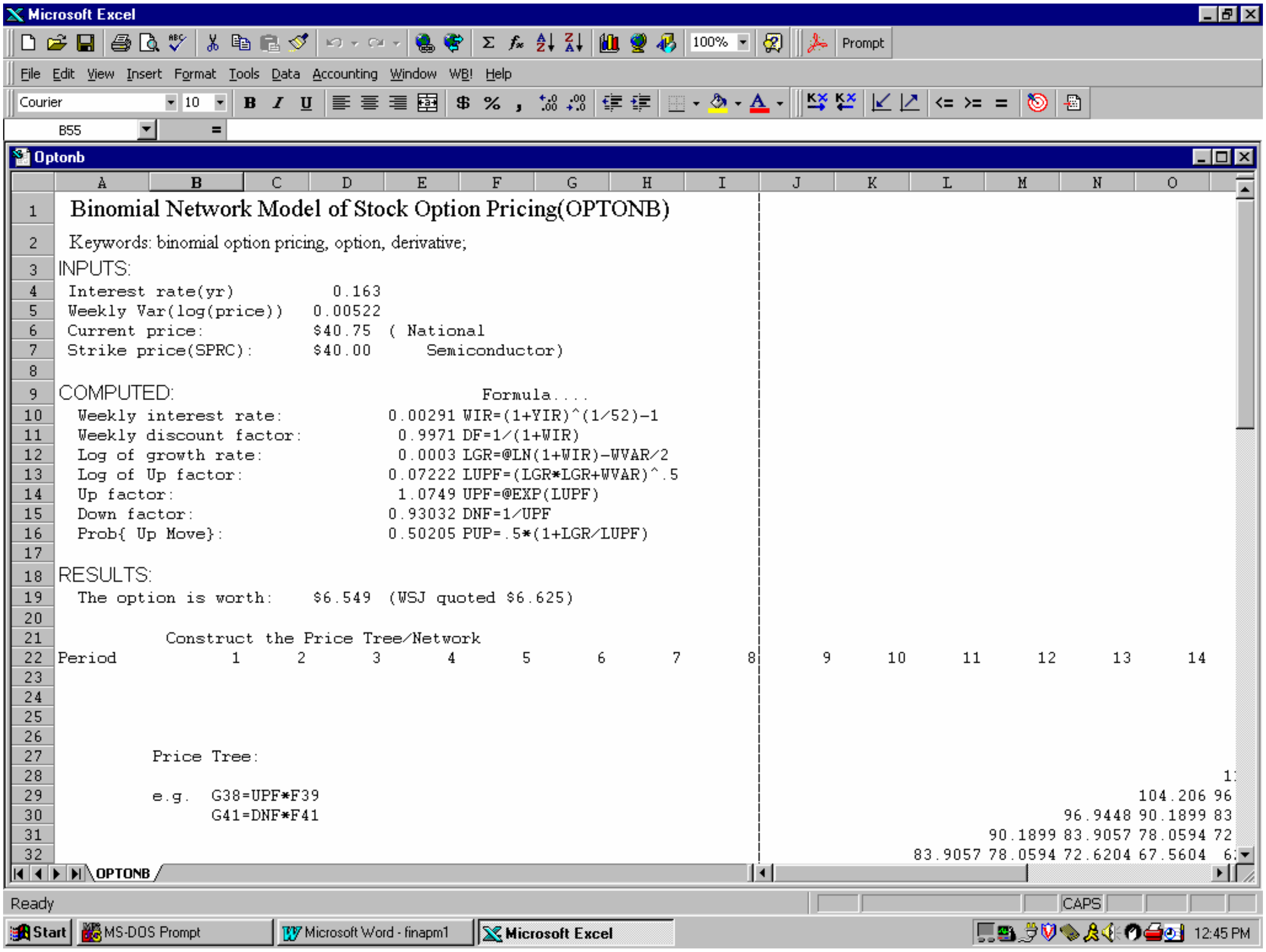
Stock prices,
Stochastic model: Black/Scholes
`Model: optonb`

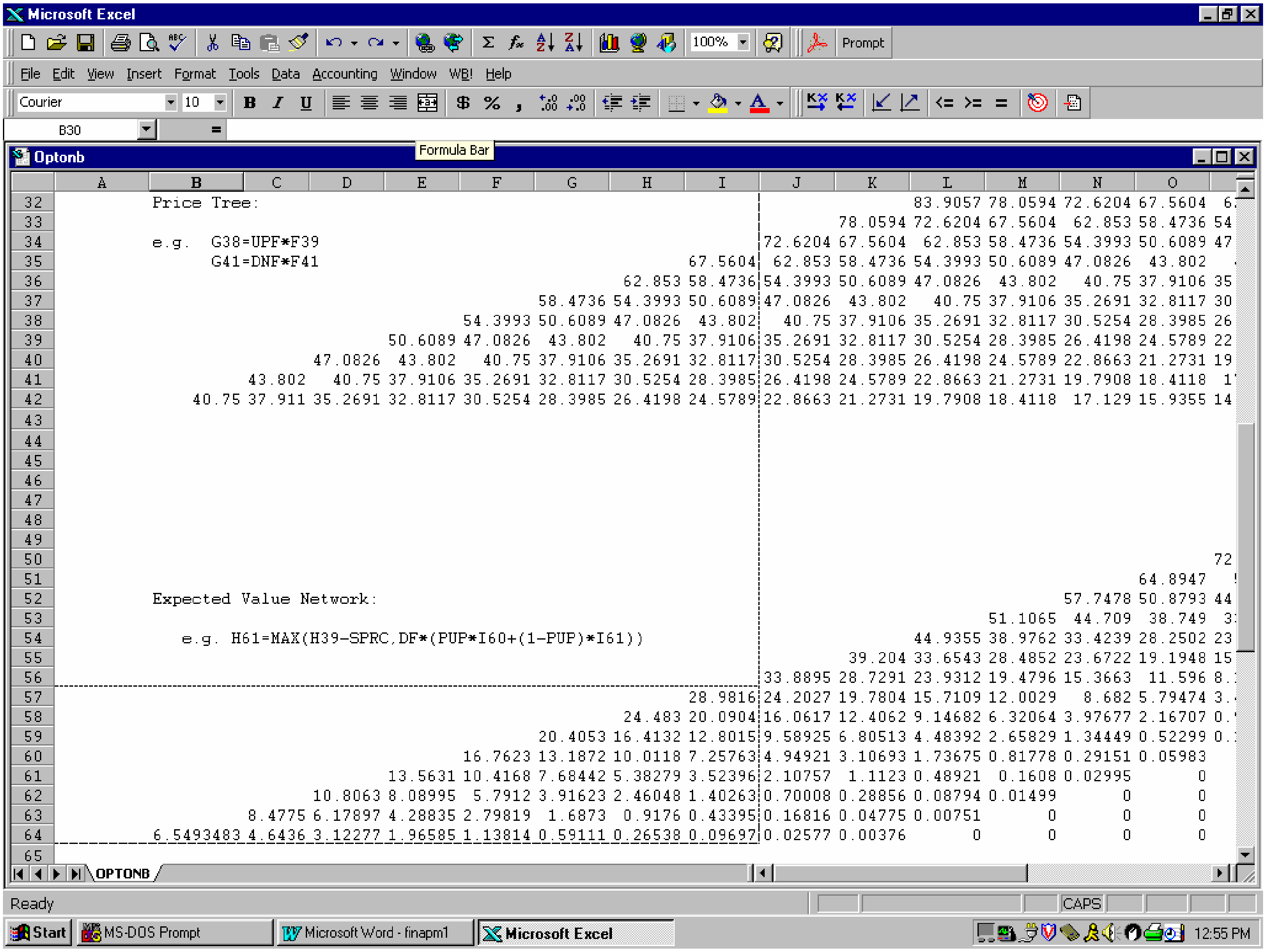
Interest rates
Stochastic model: Black/Derman/Toy
`Model:  bdtint`

Foreign exchange rates
Stochastic model: modified Black/Scholes
`Model: optonfx`

Ref.: Hull(1999)

Models: `optonb, bdtint, optonfx`


Microsoft Excel
Optonb
Binomial Network Model of Stock Option Pricing(OPTONB)
Keywords: binomial option pricing, option, derivative;
INPUTS:
Interest rate(yr) 0.163
Weekly Var(log(price)) 0.00522
Current price: $40.75 (National
Strike price(SPRC): $40.00 Semiconductor)
COMPUTED: Formula....
Weekly interest rate: 0.00291 WIR=(1+YIR)^(1/52)-1
Weekly discount factor: 0.9971 DF=1/(1+WIR)
Log of growth rate: 0.0003 LGR=@LN(1+WIR)-WVAR/2
Log of Up factor: 0.07222 LUPF=(LGR*LGR+WVAR)^.5
Up factor: 1.0749 UPF=@EXP(LUPF)
Down factor: 0.93032 DNF=1/UPF
Prob{ Up Move}: 0.50205 PUP=.5*(1+LGR/LUPF)
RESULTS:
The option is worth: $6.549 (WSJ quoted $6.625)
Construct the Price Tree/Network
Period 1 2 3 4 5 6 7 8 9 10 11 12 13 14
Price Tree:
e.g. G38=UPF*F39
G41=DNF*F41
104.206 96
96.9448 90.1899 83
90.1899 83.9057 78.0594 72
83.9057 78.0594 72.6204 67.5604 6
OPTONB
Ready
CAPS
Start
MS-DOS Prompt
Microsoft Word - finapm1
Microsoft Excel
12:45 PM


Microsoft Excel
Optonb
Price Tree:
e.g. G38=UPF*F39
G41=DNF*F41
83.9057 78.0594 72.6204 67.5604
78.0594 72.6204 67.5604 62.853 58.4736 54
72.6204 67.5604 62.853 58.4736 54.3993 50.6089 47
67.5604 62.853 58.4736 54.3993 50.6089 47.0826 43.802
62.853 58.4736 54.3993 50.6089 47.0826 43.802 40.75 37.9106 35
58.4736 54.3993 50.6089 47.0826 43.802 40.75 37.9106 35.2691 32.8117 30
54.3993 50.6089 47.0826 43.802 40.75 37.9106 35.2691 32.8117 30.5254 28.3985 26
50.6089 47.0826 43.802 40.75 37.9106 35.2691 32.8117 30.5254 28.3985 26.4198 24.5789 22
47.0826 43.802 40.75 37.9106 35.2691 32.8117 30.5254 28.3985 26.4198 24.5789 22.8663 21.2731 19
43.802 40.75 37.9106 35.2691 32.8117 30.5254 28.3985 26.4198 24.5789 22.8663 21.2731 19.7908 18.4118
40.75 37.911 35.2691 32.8117 30.5254 28.3985 26.4198 24.5789 22.8663 21.2731 19.7908 18.4118 17.129 15.9355 14
Expected Value Network:
e.g. H61=MAX(H39-SPRC,DF*(PUP*I60+(1-PUP)*I61))
72
64.8947
57.7478 50.8793 44
51.1065 44.709 38.749
44.9355 38.9762 33.4239 28.2502 23
39.204 33.6543 28.4852 23.6722 19.1948 15
33.8895 28.7291 23.9312 19.4796 15.3663 11.596
28.9816 24.2027 19.7804 15.7109 12.0029 8.682 5.79474
24.483 20.0904 16.0617 12.4062 9.14682 6.32064 3.97677 2.16707
20.4053 16.4132 12.8015 9.58925 6.80513 4.48392 2.65829 1.34449 0.52299
16.7623 13.1872 10.0118 7.25763 4.94921 3.10693 1.73675 0.81778 0.29151 0.05983
13.5631 10.4168 7.68442 5.38279 3.52396 2.10757 1.1123 0.48921 0.1608 0.02995 0
10.8063 8.08995 5.7912 3.91623 2.46048 1.40263 0.70008 0.28856 0.08794 0.01499 0 0
8.4775 6.17897 4.28835 2.79819 1.6873 0.9176 0.43395 0.16816 0.04775 0.00751 0 0 0
6.5493483 4.6436 3.12277 1.96585 1.13814 0.59111 0.26538 0.09697 0.02577 0.00376 0 0 0 0
OPTONB
Ready
CAPS
Start
MS-DOS Prompt
Microsoft Word - finapm1
Microsoft Excel
12:55 PM

Weather Derivatives/Insurance - Overview

1997: Enron and Koch agreed that Enron would pay Koch if the 97-98 Winter was cold, and Koch would pay Enron if the 97-98 winter was warm.

E.g.: B pays A $10,000 for every degree-day below 32F during December-January.

Question: What should be the premium?

Buyers: Ski resorts, road salt manufacturers, snow-blower manufacturers, construction firms.

Mainly a data fitting problem
Have 100 years of data,
Look for cycles: yearly, weekly, 11 year sunspot cycle,
El Nino cycle,
Trends: CO_2 trend, Ozone hole trend;

Flexible Spending Accounts - Overview

At start of year you may set aside a specified amount of before-tax dollars in "flexible spending" account.

During the year you may withdraw from this account to pay medical expenses not covered by regular medical insurance.

Account has a "use or lose it" nature.

How much money should be set aside?

Quick answer: Less than estimated expenses.
Longer answer: This is a standard newsvendor problem.

Model: `flexspnd`

Portfolio Repackaging/Pooling - Overview

Broker must repackage a given set of objects
E.g. mortgages, loans, into
standard bundles/securities for customers.

Example 1:
There is a single dimension, size. Each bundle should have a total size of close to $1M. Variation of bin packing

Example 2:
During the day,
Dealer accumulated units of the commodity at various prices.
Customers have bought various quantities of the commodity.

End of the day,
Dealer wants to allocate the assets explicitly to the customers
So that average price is about the same for each customer.

Difficulty: Good solutions are easy.
Large instances may be difficult to solve to optimality.

`Model: objbundl, portpart`

Market Equilibria and Pricing - Overview

Suppliers set prices, so as to maximize their revenue.

Customers decide how much to buy so as to maximize utility.

Cases:

a) Discrete, Each customer needs only 1 unit.

b) Continuous quantities

Discrete case:

Customer i purchases product $j(i)$ so that.

$ResPrice(i,j(i)) - Price(j(i)) \qquad ResPrice(i,j) - Price(j)$

Seller sets prices so as to:

Max $\sum_i Price(j(i)) - costs$,

`Model: pricprod.`

Making Spreadsheet Models Scalable- Overview

Common Problem: The real world changes rapidly.
Can we adjust/reuse our planning model just as fast?

Resolution: Two philosophies:

1) Keep unimportant details out of the model. Follow the KISS(keep it simple…) philosophy.
2) Put as much of the model in data tables as possible,
Separate the data from the model.

Some desirable features of a model:

Scalable

If we have a ten customer model, we should be able to get an eleven customer model with minimal effort. Data describing a given customer should be arranged across a single row or a single column;
Not in multiple rows, or both in rows and columns.

Parametric

If a single assumption changes, we need only change one datum. The parameter should not be hard coded throughout the spreadsheet.

Extendable.

If we have a one period model, it would be nice if we could generalize it to multiple periods with modest effort.

One style of scalability in a spreadsheet model:

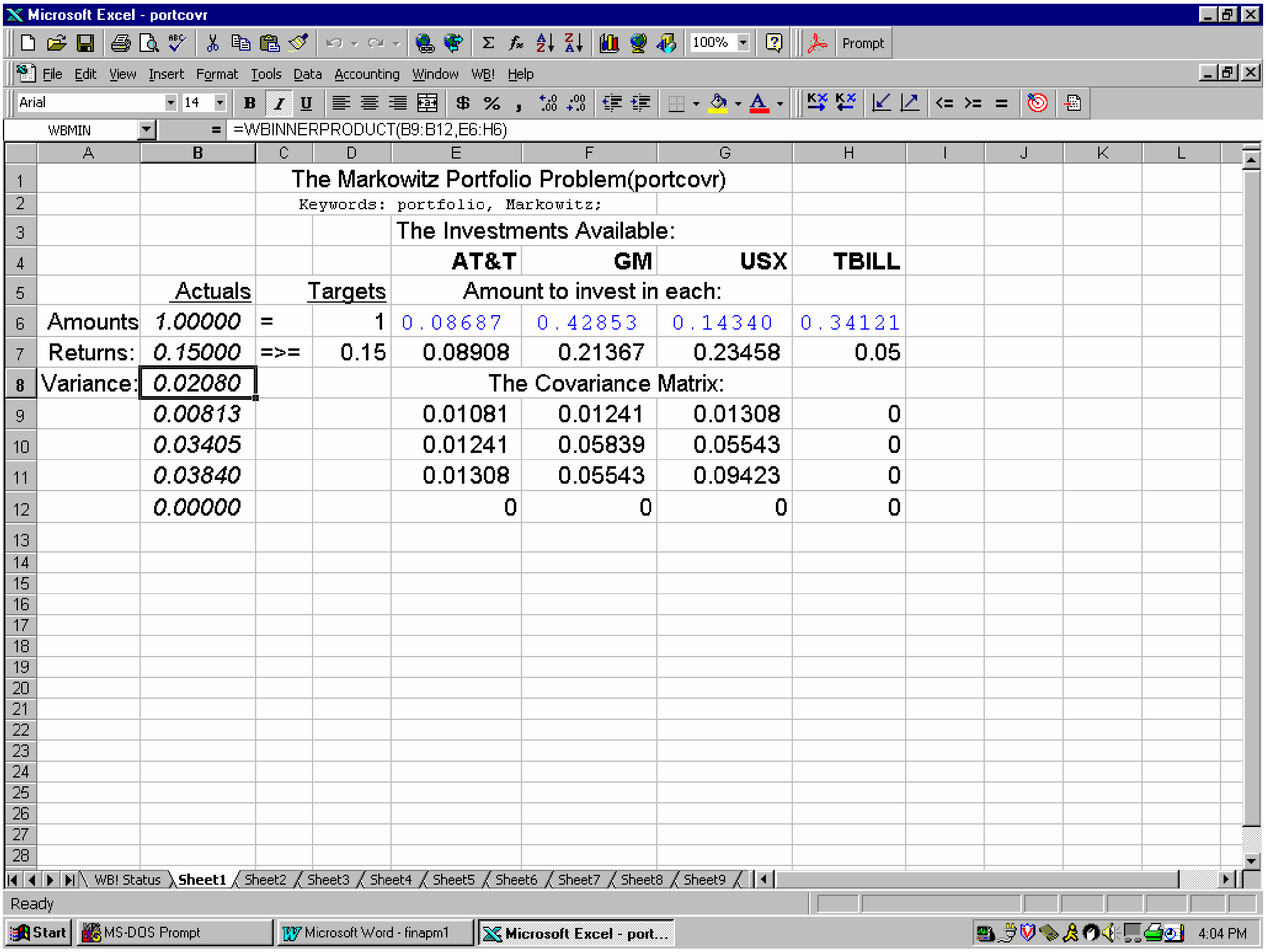
Should be able to add another product
(or source, or time period, or customer, etc.)
by doing the three steps:

1) Insert a new, empty row or column in the appropriate range.

2) Copy an existing row or column to the new row to get the formulae copied in.

3) Enter new data in data section of the new row or column.

Example



	A	B	C	D	E	F	G	H
1			The Markowitz Portfolio Problem(portcovr)					
2			Keywords: portfolio, Markowitz;					
3					The Investments Available:			
4					AT&T	GM	USX	TBILL
5		Actuals		Targets	Amount to invest in each:			
6	Amounts	1.00000	=	1	0.08687	0.42853	0.14340	0.34121
7	Returns:	0.15000	=>=	0.15	0.08908	0.21367	0.23458	0.05
8	Variance:	0.02080			The Covariance Matrix:			
9		0.00813			0.01081	0.01241	0.01308	0
10		0.03405			0.01241	0.05839	0.05543	0
11		0.03840			0.01308	0.05543	0.09423	0
12		0.00000			0	0	0	0

You can add a new investment to the above sheet by:

1a) Inserting one new row in the middle,
1b) Inserting one new column in the middle,
3a) Entering the new row data in the covariance matrix,

3b) Entering the return data and the covariance data in a column.

The formulae, *sumproduct*(), and in particular, the *wbinnerproduct*(), expand automatically, so no copying need be done to get the formulae correct.

At Home with the Range

When adding additional rows or columns to a spreadsheet, one should be familiar with how ranges are adjusted as a result of the insertion of a new row or column. If a new row is added at the end of a range, there is an ambiguity with regard to weather the new row should be part of the existing range or not. To illustrate, suppose that some cell contains the expression: =sum(a12:a36), and we insert a row with new information in row 37. Will this formula change? The answer is no. On the otherhand, if we insert a row with new information in row 35, then the formula will automatically adjust to: =sum(a12:a37). One can summarize the convention that all spreadsheet programs use in this regard with the rule that: A range will automatically <u>expand</u> if a row or column in inserted in its interior, however, it will not <u>extend</u> if a row is inserted at the end of a range. There has been at least one lawsuit by a user who made a bad decision as a result of not understanding the above rule. The rule applies to sumproduct() and wbinnerproduct(), as well as product().

Portfolios with Transaction Costs and Taxes - Details

How much difference can taxes make?

Portfolio VanS was managed without (Sans) regard to taxes.

Portfolio VanT was managed with after-tax performance in mind.

Portfolio	Distributions: Income	Distributions: Gain-from-sales	Initial Share-price	Return
VanS	\$0.41	\$2.31	\$19.85	33.65%
VanT	\$0.28	\$0.00	\$13.44	34.68%

Tax managed portfolio, probably just by chance, in fact had a higher before tax return.

It looks even more attractive after taxes. If the tax rate for both dividend income and capital gains is, say 30%, then the tax paid at year end per dollar invested in portfolio S is

$.3 \times (.41 + 2.31) /19.85 = 4.1$ cents. Whereas, the tax per dollar invested in portfolio S is $.3 \times .28/13.44 = 0.6$ of a cent.

Project Selection Portfolios - Details

Microsoft Excel - capbudx

G18 = =(E18-F18)^2

Row	A	B	C	D	E	F	G	H	I
2				Project Selection Model				(CAPBUDX)	
3				(All $ Figures in Millions)					
4		Maximize the net present value of projects selected, subject to							
5		budget constraints in the first two years,							
6		a risk constraint, and							
7		interaction constraints among projects of a logical nature.							
8		Keywords: capital budgeting, project selection, portfolio;							
9									
10			Capital Requirements		Expected	Std Dev	Variance	Logical	
11			Year 1	Year 2	NPV	in NPV	in NPV	Groups	
12	Upper Limit:		$30.00	$16.00		5	25	1	0
13	Constraint test:		>=	>=			>=	=>=	=>=
14	Actual:		$28.00	$15.00	$48.00	$3.74	14	1	0
15									
16	Project					Low End			
17	Name	DO IT?				NPV(p=.15)		A	B
18	Advertise X	1	$5.00	$2.00	$20.00	$18.00	4		1
19	Install X	1	$21.00	$3.00	$17.00	$16.00	1		-1
20	Refurbish99	0	$9.00	$0.00	$12.00	$9.00	9	1	
21	Refurbish00	1	$2.00	$10.00	$11.00	$8.00	9	1	
22	Training	0	$5.00	$0.00	$8.00	$6.00	4		
23	R&D	0	$7.00	$1.00	$5.00	$4.50	0.25		
24	End Guard	0	0	0	0	0	0		

Multi-Factor/Scenario Models - Details

There are several factors, e.g. exchange rate, R&D success.
Several scenarios are possible for each factor.
A complete realization corresponds to
one scenario occurring for each factor,
e.g., exchange rate was 122:1, R&D was successful.

Define:

r_{iks} = return of asset *i* due to factor *k* if scenario *s* occurs (for factor *k*),

p_{ks} = probability that factor *k* outcome is scenario *s*,

x_i = amount invested in asset *i*,

r_{ik} = expected return of asset *i* of from factor *k*,

$$= \sum_s p_{ks} r_{iks}$$

R_{ks} = return of the portfolio due to factor *k* under scenario *s*,

$$= \sum_i x_i r_{iks}$$

r_k = expected return of portfolio due to factor *k*,

$$= \sum_i x_i r_{ik}$$

σ_k^2 = variance of portfolio due to factor k,

$$= \sum_s p_{ks} (\sum_i x_i r_{iks} - r_k)^2$$

$$= \sum_s p_{ks} (R_{ks} - r_k)^2$$

Z^2 = variance of portfolio,

$$= \sum_k \sigma_k^2$$

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