Application Survey Paper

Electrical Generation Unit Commitment Planning

The electrical unit commitment problem is the problem of deciding which electricity generation units should be running in each period so as to satisfy a predictably varying demand for electricity. The challenge is illustrated in Figure 1. In short range planning, a period might be 15 minutes to one hour. The problem is interesting because in a typical electrical system there are a variety of units available for generating electricity, and each with its own characteristics. At one extreme, a nuclear power unit can provide electricity at a very low incremental cost for each additional megawatthour of energy, but it has both a high cost of starting up again once it has been shut down and it takes awhile to bring it back up to full power. A typical nuclear unit may be shut down only in the Spring or Autumn, when there is very little heating or air-conditioning demand, so demand is lowest. At the other extreme, a gas turbine generator can be started up in a few minutes, however, its incremental cost per megawatthour is much more expensive.

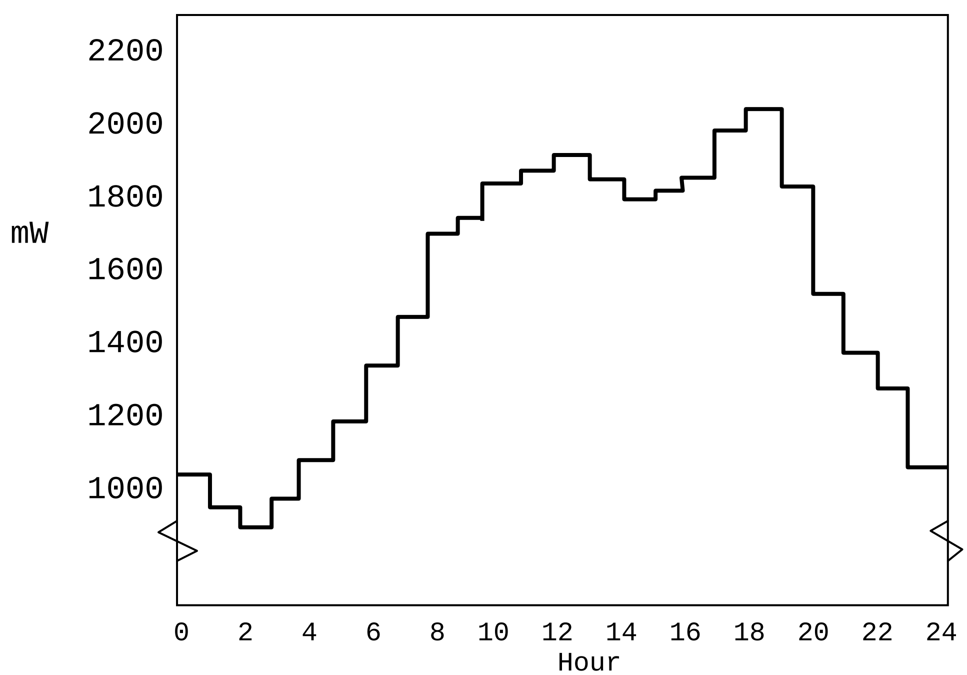

Figure 1. Typical demand curve for electricity by hour.

The obvious policy is that as demand increases we first turn on the efficient but costly to start generators and lastly turn on the least efficient by cheap to start. As demand decreases, we shut down units in the reverse the order. Decisions are a little more interesting if there is a modest peak in demand of short duration. Then it may be economic to skip an intermediate unit and instead turn on an inefficient but cheap-to-start unit for the duration of the short spike. Various other features of various types of units also complicate the decision. We partition generator units into four types in order of the complexity of their features.

A complete version of a simple unit commitment model can be found in the LINGO library in the model `unitcom1.lg4`. Like most operational models, this model can be used in two ways: a) to actually schedule daily operation, and b) to do long range investment planning via what-if analysis, in which we "plug-in" various future demand and equipment scenarios and observe resulting daily cost of operation.

From a management perspective, there are at least four different types of generating units that one might find in a system. We describe each type below.

Conventional Units

We classify nuclear, coal, oil, and gas-fired units as conventional units. Gas fired units come in two flavors. The simplest is essential a jet engine with an electrical generator hooked to its shaft. A "combined cycle" gas fired unit additionally produces electricity by using the hot exhaust gas to convert water to steam and generate electricity by sending the steam through a steam turbine. All of these types, from a day-to-day economic perspective can, to a good first approximation, be described by the following data:

CS_i = cost of starting up generator i ,
CF_i = fixed cost of having generator i running(or "spinning") during a period,
CG_i = cost of generating one megawatt of energy from unit i,
UL_i = upper limit on number of megawatts of power generated by unit i in each period.

The data that are driving the whole process are:

D_t = megawatthours of energy demanded in period t,

The decision variables are:
x_{it} = megawatthours of energy generated by unit i in period t,
y_{it} = 1 if unit i is running or "spinning" in period t,
z_{it} = 1 if unit i is starts to run in period t,

The startup cost factor may include the fact that the generator is not as efficient during startup and that the emission of pollutants such as sulfur oxides may be higher during startups. In words, the model at this point is:

Minimize
startup costs + running costs + energy generation costs over the planning horizon,

s.t.
For each period t:
Amount of energy generated = the demand in period t,

For each period t and generator i:

The amount generated in the period does not exceed the generator's capacity,
A startup cost is incurred if the generator starts up, and
A running cost is incurred if the generator is running or spinning.

Run-of-River, Wind, and Tide Based Generation

For some power generation units, the amount of power that can be generated depends upon the time of day, year, etc. The simplest example is generation based on the change of water level due to tide at a sea coast. Tidal power units exist in France, on the Bay of Fundy in Canada, and in China. Other examples are wind generated power where the amount of power available depends upon the amount of wind, a factor that may vary in a predictable way over the period of a day or year. Substantial wind generated capacity exists in several states, such as California. Some hydro power units are located on rivers but without a significant storage reservoir. Thus, the power generation capacity is proportional to the current flow in the river. Such a unit is called run-of-river hydro. For these kinds of units, there is not a single capacity that applies to every period, but rather a capacity for each period, represented as:

U_{it} = upper limit on the number of megawatthours of energy generated by unit *i* in period *t*.
You can get up to U_{it} units of energy from unit *i* in period *t*. Any unused capacity is lost forever.

Hydro Based Generation

About 15% of the world's electricity comes from hydro-power, that is, electricity generated by falling water. A distinctive feature of a (non run-of-river)hydro power unit is that (potential) energy can be stored. That is, there is a large water reservoir, or pond, behind a dam. Unless the pond is full, so that additional inflow must be spilled, water that flows into the pond in one period may be stored to be used later to generate electricity. The following additional data are required to describe a hydro unit:

FLO_i = amount of potential energy flowing into the pond each period,
IMX_i = maximum amount of energy that can be stored.

We need the additional decision variables:

I_{it} = amount of potential energy stored in the pond of unit *i* at the end of period *t*.
$SPILL_{it}$ = amount of water, measured in energy, spilled from the pond of unit *i* in period *t*, without generating any electricity.

Thus for each period *t* we have the inventory equation:

$$I_{it-1} + FLO_i = I_{it} + x_{it} + SPILL_{it} ,$$

and the limit:

$I_{it} \leq IMX_i$.

If the model is short term, e.g., a day or a week, then the generating cost for hydro should probably contain an opportunity cost component to represent the future value of hydro energy. For example, we might not want to use hydro today instead of coal fired energy, if in several weeks we will run out of hydro and will have to use expensive gas fired energy where we might instead used stored hydro.

A second-order effect that we are disregarding is that as water is allowed to accumulate in the reservoir, the height or "Head" will increase. The amount of power you can generate by releasing water from a reservoir is proportional to the Head by the approximate formula:

Power(in kW) = 5.9 * Release_rate(in m^3/sec) * Head(in m).

Pumped Storage Generation

Pumped storage is an extension of hydro in which there is a reservoir at both the bottom and the top. During low demand periods(e.g., at night), cheap electricity(e.g., from a nuclear generator or night time wind or tide) can be used to pump water from the lower reservoir to the upper reservoir. There are about 40 pumped storage facilities in the U.S. A pumped storage facility is described by the same types of parameters and decision variables as a hydro unit, plus the additional parameters:

pf_i = number of megawatthours of energy required to pump one megawatthour of energy into the upper reservoir of i from the lower reservoir,

pmx_i = maximum number of megawatthours of energy that can be pumped into the upper reservoir of i in one period.

An additional variable is needed:

w_{it} = megawatthours of water energy pumped in reservoir i in period t.

The inventory balance equations for the reservoir must be generalized to:

$I_{it-1} + FLO_i + w_{it} = I_{it} + x_{it} + SPILL_{it}$,

$w_{it} \leq pmx_i$.

The term $pf_i * w_{it}$ must be added to the energy demand equation as a use of energy.

Other Considerations

The simple model above is a starting point for a variety of practical generalizations. For example:

Multiple markets: There may be multiple markets with the possibility of shipping electricity among the markets.

Pollution restrictions: Pollutants such as sulfur dioxide, nitrous oxides, ash, and carbon dioxide are undesirable. Some generators pollute more than others and there may be limits on how much total pollution may be generated in total in a period or under certain conditions.

Uncertainty and Spinning Reserve: Demands are not known with certainty. Generation units may unexpectedly fail. One reaction to this is to require a certain amount of additional capacity to be quickly available (or spinning) in a period so that additional power is quickly available if by chance it is needed. Providing this backup is sometimes referred to as ancillary services.

Ramp-up features: One may have a "ramp up" constraint that constrains the rate at which output can be increased for a generator. E.g., it might look something like:

$$x_{it} \leq 1.15* \; x_{it-1} + 20;$$

Each generator may have a minimum output level above which it must operate if it is operating at all;

The startup cost may depend upon how long the unit has been shutdown;

Nonlinear Conversion: In hydro units it may be that the amount of electrical power produced is a nonlinear function of the cubic feet/second of water released through the turbine. For example, a plausible tabulation of mWh of electricity produced as a function of million cubic feet(mcf) of water per hour released through the turbine might look as follows:

mcf	mWh	Incremental wH/cf
0	0	
4	18	4.5
8	49	7.75
11	84	11.67
13	95	3.67

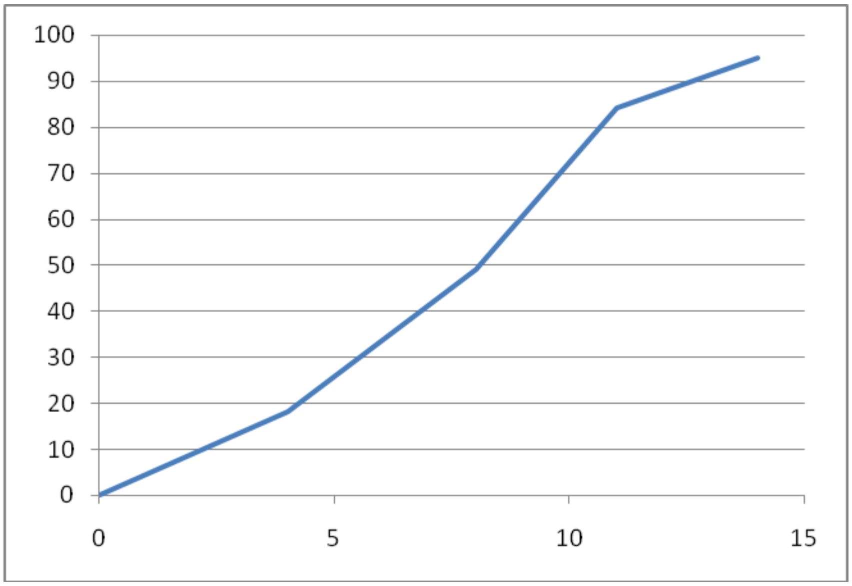

Figure 2. Output in mW(vertical) vs. flow in mcf(horizontal).

For this turbine, its "sweet spot" is from 8 to 11 mcf per hour or from 49 to 84 mWh per hour where the conversion rate is 11.67 mWh per mcf of water through the turbine. In a model this would be represented as a piecewise linear curve.

Linked Reservoirs: A system may have several hydro units where the outflow of one is input into the next. Thus, if the downstream pond is low, it may be useful to run an upstream unit at a higher rate than one might otherwise run it just considering local conditions. If the downstream pond is close to full, you might want to run the upstream unit at a lower rate than one would otherwise. This is modeled by making the outflow in period *t* of the upstream unit part of the inflow to the downstream unit in some future period *t* + *k*.

Example

Here is a sample data set. Look at it and try to answer the questions that follow it.

```
DATA:
 GENR =   COAL1  GAS1 GAS2 WIND1 HYDRO1 PUMP1;! The units;
  CF  =    2775  5980 5290  1     2.0    1.9; ! Fixed cost/period if spinning;
  CG  =      14    22   25  1     1.1     2;  ! Cost/MWH generated;
  CS  =   90000 12900 8000 1700   800    790;! Cost to start spinning;
 MXG  =    1100   760  680  600   900    660;! Max gen capacity in period;
 INITY  =     0     0    0    0     0      0; ! Initially spinning or not;
 PERIOD = 1..12;
  DEM =    900 1790 2380 2490 1520 1298 2200 1030
          1900 2360 1900 1790; ! Demand in MWH;

 ! Units with variable capacity;
  VGEN = WIND1;   ! Capacity in each period;
  VMXG =    180   210   270   280 310   200 300 250
            150   100   100   100;
```

```
 ! Hydro and pumped storage units;
  HNP = HYDRO1 PUMP1;! Name of the hydro and pumped units;
  FLO =  220     0;  ! Inflow per period;
  IMX = 1000   970;  ! Max inventory level;
  INI =   200    0;  ! Initial inventory;

 ! Pumped storage;
  STORE = PUMP1;
     PF = 1.3; ! MWH needed to put 1 MWH in storage;
    PMX = 320; ! Max pumped into storage/period;
ENDDATA
```

Questions:

1) The `WIND1, HYDRO1,` and `PUMP1` units have the lowest cost/MWH, as well as the lowest cost to start up. Would therefore you run them at full capacity every period?

2) We clearly need the capacity of the big COAL1 unit in some of the high demand periods. In which periods would you run COAL1 at full capacity?

3) As demand builds, e.g., over the first four periods, will you add capacity in order of increasing cost/MWH as demand increases? E.g., as COAL1 runs out of capacity, then add GAS1, and when GAS1 runs out of capacity turn on GAS2?