

Formulating SP\ Stochastic Programming\ Scenario Planning Models in What's*Best*!

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Is there a general way of incorporating probabilistic uncertainty into optimization problems?

Yes, goes by the name, "Stochastic Programming(SP)".

Can also perhaps more suggestively think of it as Scenario Planning(SP).

Basic idea is to use a finite number of scenarios, each with a specified probability.

May have a multi-period sequence of random events.





If uncertainty is a significant factor:

- Simple deterministic analysis may suggest a solution far from optimal, e.g., stocking to exactly meet expected demand may miss the high profit of occasional really high demand.
- Simple scenario-by-scenario analysis, may miss the optimal solution, e.g., the <u>solution that is optimal</u> when all scenarios are taken into account <u>may not be optimal for any single scenario</u>.
- 3) Simple expected value analysis, even if it takes into account uncertainty, may miss the fact that we really care about the distribution of outcomes, e.g., the low probability but catastrophic outcome.
 <u>SP optimization supplies you with the distribution of outcomes</u>.
 You may have two or more random variables with the same mean and standard deviation, but dramatically different distributions...



Perhaps We Should Be Concerned About the Distribution...



Here are the histograms of three random variables, each with *Mean*= 64, *SD*= 8.



0.14 0.12 Frequency 0.1 0.08 0.06 0.04 0.02 0 61.201376 65.80402 68.097315 12.697019 15.00272 54.3019 46.⁵⁹⁸⁸⁵⁵³ 58.900779 63:594589 10.396288 51.996821 Outcome 3

Histogram



Multi-Stage Decision Making Under Uncertainty

Stochastic programming, or Scenario Planning, or SP for short, is an approach for solving problems of multi-stage decision making under uncertainty. SP is designed to solve problems of the following form:

0) In stage 0 we make some decisions, taking into account that later,

- 1) At the beginning of stage 1, "Nature" makes a random decision,
- 1a) At the end of stage 1, having seen nature's decision, as well as our previous decisions, we make some decisions, taking into account that ...
- 2) Somewhat later at the beginning in stage 2, "Nature" makes a random decision,
- *n*) At the beginning of stage *n*, "Nature" makes a random decision, and *n*.a) At the end of stage *n*, having seen all of nature's *n* previous decisions, as well as all our previous decisions, we make a decision,

If there are only a finite number of outcomes(which is true computationally) for nature at each stage, then it may be helpful to visualize the process by a tree.



Viewed as a Tree...

Notation:



Applications of SP, Some Examples

- + Financial portfolio planning over multiple periods for insurance and other financial companies, in the face of uncertain prices, interest rates, exchange rates, and bankruptcies,
- + Capacity and Production planning in the face of uncertain future demands and prices,
- + Fuel purchasing when facing uncertain future fuel demand and prices,
- + Optimal exploration planning for petroleum companies,
- + Foundry metal blending in the face of uncertain input scrap qualities,
- + Fleet assignment: vehicle type to route assignment in the face of uncertain route demand,
- + Electricity generator unit commitment in the face of uncertain demand,
- + Hydro management and flood control in the face of uncertain rainfall,
- + Optimal time to exercise for options in the face of uncertain prices,
- + Product planning in the face of future technology uncertainty,
- + Revenue management in the hospitality and transport industries.



Some generic but common two stage (0 and 1), examples:

Example 1: Capacity Planning (Multi-dimensional Newsvendor) Stage 0, decisions:

 x_i = capacity installed of type *i*; made before seeing demand, Stage 1 beginning, random events observed:

 d_{sj} = demand for product type *j* in scenario *s*, for *s* = 1, 2,..., *ns*, Stage 1 end:

 y_{sii} = amount shipped from *i* to *j* if scenario is *s*;

Model:

 $Max = -\sum_{i} c_{i} x_{i} + \sum_{s} \sum_{i} \sum_{i} r_{ii} y_{sii} / ns; \qquad ! \text{ Assumes all scenarios equally likely;}$

For each scenario *s* and source *i*:

$$\Sigma_j y_{sij} \leq x_i;$$

 $\Sigma_i y_{sii} \leq d_{si};$

! Capacity constraints;

For each scenario s and demand type *j*: ! Demand constraints;



2) Portfolio planning.

Stage 0, decisions:

 x_i = amount invested in instrument *i*; Stage 1 beginning, observe random outcomes: r_{si} = return on investment in instrument *i* in scenario *s*,

for *s* = 1, 2,..., *ns*,

Stage 1 end:

 y_s = return of portfolio if scenario is s,

 u_s , d_s = deviation up, down of return from target;

Model:

 $\Sigma_i x_i \le 1$; ! Compute Budget constraint;

For each scenario *s* :

 $y_s = \sum_i r_{si} * x_i$; ! Compute scenario return; $u_s - d_s = y_s - target$; ! Compute deviations from target;

 $\Sigma_s y_s /ns \ge target;$! Expected return achieves target, all scenarios equally likely; $Min = \Sigma_s d_s /ns;$! Min downside risk; LINDO SYSTEMS INC.

Plant configuration decisions, e.g., General Motors Had too much capacity.

Needed to close or refocus an unknown number of plants.

Investment Portfolios at Insurance Companies,

e.g., Yasuda-Kasai in Japan.

Had been using Markowitz "mean-variance" portfolio optimization. Markowitz assumes risks have a Normal distribution(symmetric) Actual risks were too non-symmetric (This is insurance)



Multi-Stage Tree Structures in Practice...

General Motors used a 5 period, (but 2 stage) model:Periods 1-4: The next 4 years,Period 5: Year 5 and out to infinity modeled using present values.

Plant reconfiguration <u>decisions</u> were made only at <u>beginning</u> of year 1. No reconfiguration decisions thereafter.

General Motors historically made three forecasts, with associated probabilities, for each year, into the future.

StageBranchesRepresents1 $3^5 = 243$ Next 4 years + infinity

Total number of full scenarios = 243.



+ Downside risk

+ Unsatisfied demand for a product transfers to other products according to a substitution matrix. One dozen products.

+ Infinite final period.

Key parameters:

 $c_{pv} = \text{cost per unit to produce vehicle } v \text{ in plant } p \text{ (only possible if plant is open),}$

- τ_{vw} = fraction of unsatisfied demand for vehicle *v* that transfers to vehicle *w*, (from surveys),
- $CAP_{p\sigma}$ = capacity of plant *p* in configuration σ ,

Key variables:

 x_{spv} = number of units of vehicle v produced in plant p in scenario s.



The key constraints in words are:

For each scenario *s*

For each vehicle *v*:

 $Production_{vs} + Unsat_{sv} = Demand_{sv} + Transfer_in_{sv};$

For each vehicle v and w: $Transfer_from_to_{svw} \leq \tau_{vw} * Unsat_{sv};$

For each plant *p* and configuration σ : $Total_production_{sp} \leq CAP_{p\sigma} * y_{p\sigma}$





$penalty_s \ge threshold - profit_s$;

Expected downside risk constraint:

 $\sum_{s} Prob_{s} penalty_{s} \leq tolerance;$

Both threshold and tolerance are parameters.



General Features:

Two stages,

Stage 0, make purchase and storage decisions,

Stage 1: Ten scenarios, corresponding to ten previous representative weather patterns, scaled up to today. Each scenario has 365 periods.

Storage costs are nonlinear, first units are easy to pump in, last units require much energy to pump in.First units withdrawn can be withdrawn rapidly, last units can be withdrawn only slowly.

Contracts have daily min and max and total over all days.



Doing SP in either What's Best! or LINGO

Essential Steps:

- 1) Write a standard deterministic model (the core model) as if the random variables were variables or parameters.
- 2) Identify the random variables, and decision variables, and their staging.
- 3) Provide the distributions describing the random variables, [Why separate (2) and (3) ?]
- 4) Specify manner of sampling from the distributions, (mainly the sample size), and
- 5) List the variables for which we want a (What's *Best*! only) scenario by scenario report or a histogram

How is SP Information Stored in the SpreadSheet?

All information about the SP features is stored explicitly/openly on the spreadsheet.

- Core model is a regular deterministic What's*Best*! or LINGO model. You can plug in regular numbers in a random cell to check results.
- 2) Staging information is stored in Decisions: WBSP_VAR(stage, cell_list) and Random variables: WBSP_RAND(stage, cell_list);
- 3) Distribution specification is stored in WBSP_DIST_distn(table, cell_list); where distn specifies the distribution, e.g., NORMAL cell.
- 4) Sample size for each stage is stored in WBSP_STSC(table);
- 5) Cells to be reported are listed in WBSP_REP(cell_list) or WBSP_HIST(bins, cell); LINDO SYSTEMS INC.



The "Core Model" is a completely valid Excel model.

If you are doing neither simple optimization nor SP, you can do complete "What-If" analysis with it as a valid deterministic model.

If you have not turned on SP, you can do simple optimization with it like any deterministic What'sBest model.



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28	b) Cells designated as stage 1 random variables(of	some spe	cified distributio	on) are repla	811,812,813,822		Cancel	F					
29	c) the behind the scenes objective is to maximize ne	et profit av	eraged over all	scenarios.	List of selected cells (sele Add or Remove):	ct and chan	ige by						
30			-		'Model'!B11		Add						
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Input via a Dialog Box, Setting Various Options

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Α	В	C L) <u>E</u>	FGH		J	K	L	M	N	0
Stochastic/Scenario Optimization of N	ewsvend	dor Problen	n in Wha	t'sBest (Li	near ve	ersion)					
Given all costs and prices, in											
Stage 0 we must decide how many newspapers to	stock. In										
Stage 1, in the beginning, unknown demand is reve	aled to us, a	and finally in									
Stage 1, at the end, we compute our sales and the	resulting pr	ofit.									
1) Core model:											_
CP = Purchase cost/unit	= 30										_
H=Holding cost/(unit leftover)	= 10					Add stochastic	data here				_
P=Shortage cost/(unit unsatisfied demand)	= 5				2) Sta	age information	1				
V=revenue per unit sold	= 60					WBSP_VAR	(Sisa	stage 0 de	cision)		
S=Stock level(stage 1 decision)	58 034	<<== Stage 0	decision			2		3) Distr	ibution in	formatio	n
D=Demand(stage 2 random variable)	85 151	<<== Stage 1	random dei	mand		WASP RAND		WRSP	DIST NO	RMAL	·
I S= Lost sales	= 27 117	<<== Stage 1	(recourse)	decision				W001_		Mean de	emand
ILSGE1 LS >= D - S (constrain		<c== 1<="" stage="" td=""><td>constraint</td><td></td><td></td><td>WASP VAR</td><td></td><td></td><td>2</td><td></td><td></td></c==>	constraint			WASP VAR			2		
	,	Side 1							2	J.D.	
[IDEF] I=Inventory=S-D+LS	- 0	<<== Stage 1	decision an	a constraint.	0.0						
) =>=	<<== Stage 1	non-negativ	ity constraint.	4) Sa	mpie size	Stage Scenario	IS			
[TCDEF] TC = Total cost of goods = CP * S	= 1741	<<== Stage 0	cost compu	itation.		WBSP_STSC					
[THDEF] TH = Total Holding cost=H*I	= 0	<<== Stage 1	holding cos	t computatior			1 5	0			
[TSDEF] TS = Total Shortage cost= P*LS	= 135.58	<<== Stage 1	shortage co	st computation	on.			Stochastic Solv	er Options		X
[VIDEF] $VI = Revenue = V^*(D-LS)$	= 3482.1	<<== Stage 1	revenue co	mputation				-			
Profit, expected value, [To be maximized]	=	elage .		inp attation.	5) Re	porting cells		I Use Stocha	stic Modeling Sup	port	
[TPDEF] TP = VI - TC - TH - TS	1605.449	<<== Stage 1	expected v	alue (maximiz	e)	WBSP_REP		Optimization	is Method:		Solver Decides 💌
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Overview:								Common Size	per Stage:		
The user enters only a generic scenario 1.								Complete		hikuting Only	2
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a) Cells designated as stage 0 decision variables a	re constrair	ned to be equal	in all scena	rios,				Expected	Value of Wait-an	d-See Model's Ob	ojective
 b) Cells designated as stage 1 random variables(o 	t some spe	cified distributio	on) are repla	ced by a rand	om varia	ble in each scer	nario	Expected	Value of Policy B	ased On Mean O	utcome
c) the penind the scenes objective is to maximize r	iet protit ave	eraged over all	scenarios.					Expected	Value of Perfect	Information	
								Expected	Value of Modelin	Uncertainty	
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Setting Retention:

Any settings made with a dialog box are retained when the workbook is saved. The same settings will be there when the workbook is next re-opened.

Settings such as Adjustable cells, constraints can be found by clicking on: Add-Ins | WB! | Locate



Standard Scenario Report, One Line/Scenario

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38	- 15-	78 305639	59 770935	14.174015	22/0.23303		
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41	- 19-	78.305639	111.322138	33.016499	2184.086669		
42	- 20-	78.305639	74.572581	0	2087.855151		
43	- 21-	78.305639	39.855992	0	-342.306132		
44	- 22-	78.305639	93.293771	14.988132	2274.228504		
45	- 23-	78.305639	78.964977	0.659339	2345.87247		
46	- 24-	78.305639	52.418108	0	537.042035		
47	- 25-	78.305639	59.609509	0	1040.440057		
18	- 26-	78.305639	53.327633	0	600.708757		
49	- 27-	78.305639	100.794988	22.48935	2236.722415		
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Newsvendor with Normal Demand



Even though the driving random variable, Demand, has a symmetric distribution, why is the output, Profit, so skewed?



The Generic Capacity Planning Under Uncertainty Model

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(The Generic) Capa	city Plann	ning unde	er Uncer	tainty									
Stage 0: We decide	what capac	ities to ins	stall at va	rious su	pply plac	es(invento	ories, technologies,	, etc.).					
Stage 1, Beginning:	Demands a	t various d	demand lo	ocations	are reve	aled,							
Stage 1, End: We sa	tisfy demar	nds from a	vailable o	capacitie	s (by sol	ving a tran	sportation problem	ı).					
Step 1: Core Model	Cost/unit	Capacity		Upper			Step 2a: Staging in	nfo					
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j A mita	Amount	snipped		<u>10181</u>	capacity		Step 5. Scenario/S						
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Electra	U	U	400	400	=<=				1	10			
Generic backup	33	0	33	66	<=		Step 4: Reporting	Info					
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Demands(random):	333	383	433										
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Capacity Planning Under Uncertainty, Scenario Profit



Capacity Planning, Scenario by Scenario Report

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Plant Location with Random Demand

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2	Stage 0: We de	ecide whi	ich plant	s to (ke	eep) o	pen, each	with a presp	ecifie	d	capacity									
3	Stage 1: Begin	ning, De	mands a	t variou	us loca	ations are	revealed,												
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5									2)	Staging Info								1999 (P	
6	1) Core Model	Fxd Cost	Capacity	Open?	Effecti	ive Capacity				WBSP_VAR	Declare	the stage 0 d	decision	S					
7	Atlanta	90	59	0	0					WBSP_RAND	Declare	stage 1 rand	lom vari	ables					
8	St.Louis	64	65	1	65					WBSP_VAR	Declare	stage 1 deci	sions						
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The output tab,

WB!_Stochastic, contains two types of information:

- 1) Various expected values that measure the cost of uncertainty,
- 2) A scenario by scenario listing of selected variables so we can explicitly verify what happens in each possible scenario.

We may optionally also generate histograms in a WB!_Histogram tab.

Later, we will discuss the various expected values and the various costs of uncertainty.



Plant Location, Scenario Report

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Multi-Stage Portfolio Model with Downside Risk

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Multi-Stage Portfolio Model with Downside Risk

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Multi-stage Portfolio: Solution and Policy

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Terminal Wealth Distribution: College/Retirement Planning


Yield Management: Bird in Hand vs. Future Bird in Bush

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Yield Management: Report and Policy

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Stopping Problem Solution and Policy

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21	4	-0.08	88.059	1	10.941		WBSP	VAR		WBSP RA	ND		WBSP DI	ST DISCR	ETE SV					
22	5	0.09	95.985	0	0.000		WBSP	VAR		WBSP RA	ND		WBSP DI	ST DISCR	ETE SV					
23								- and a second		_				_						1
24		Number ti	imes sold:	1			3) Sa	mple siz	es											
25	C	an sell at m	nost once:	=<=			WBSP	STSC			4) Repo	rting	cells							
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Readv	WBI Star	LUS / WB!_Stocha	suc Model / W	VBI_HIST													mam	130%		Ŧ

Put-Option, 60% of Time Does Not Pay Off





Put Option, Scenario Detail

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0			PRICEO	SELLO	PV0	PRICE1	SELL1	PV1	PRICE2	SELL2	PV2	PRICE3	SELL3	PV3	PRICE4	SELL4	PV4	PRICE5	SELLS	
2			SIAGE U	SIAGE U	STAGE 5	STAGE I	STAGE I	STAGE 5	STAGE 2	STAGE 2	STAGE 5	STAGE 3	STAGE 3	STAGE 5	STAGE 4	STAGE 4	STAGE 5	STAGE 5	STAGE 5	
3	- 1-		100	Ω	9 72053	92	0	10 0121	93 84	0	10 3125	95 7168	0	10 6219	88 05946	1	10 9405	95 98481	0	-
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5	- 3-		100	0	9.72053	92	0	10.0121	93.84	0	10.3125	95.7168	0	10.6219	88.05946	1	10.9405	81.0147	0	
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3	- 11-		100	0	2.60093	92	0	2.67896	93.84	0	2.75933	102.286	0	2.84211	94.10275	0	2.92737	95.98481	1	
4	- 12-		100	0	10.7183	92	U	11.0399	93.84	U	11.3711	102.286	U	11.7122	94.10275	U	12.0636	86.57453	1	
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6	- 24-		100	0	11.5923	92	0	11.9401	93.84	0	12.2983	86.3328	1	12.6672	88.05946	0	0	81.0147	0	
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2	- 30-		100	0	13.5357	92	0	13.9417	84.64	1	14.36	86.3328	0	0	79.42618	0	0	73.07208	0	
3	- 31-		100	0	13.5357	92	0	13.9417	84.64	1	14.36	86.3328	0	0	88.05946	0	0	95.98481	0	
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7	- 35-		100	0	13.5357	92	0	13.9417	84.64	1	14.36	86.3328	0	0	94.10275	0	0	95.98481	0	
8	- 36-		100	0	13.5357	92	0	13.9417	84.64	1	14.36	86.3328	0	0	94.10275	0	0	86.57453	0	
9	- 37-		100	0	13.5357	92	0	13.9417	84.64	1	14.36	92.2576	0	0	84.87699	0	0	92.51592	0	
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DEA: An SP Application with No Randomness

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1	DEA a	sa	Scenario Pl	anning Pro	blen	<u>n</u>											
2	Which s	choo	Is efficiently co	onvert their inp	uts int	o outputs?											
3	1) Core	Mod	el for one sch	lool						Weights	on Inputs an	d Outputs					
4	Case=	6		(Selected outp	out)	Selected input			Min wgt	0.0203444	0.0005	0.001682	0.004311				
5			Efficiency=	1		1	=	1	0.0005	>=	=>=	>=	>=				
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9		1	Bloom	1.0038069	<=	1.85073168				89.39	64.3	25.2	223				
10		2	Homewood	1.28477828	<=	1.804200413				86.25	99	28.2	287				
11		3	New Trier	1.4161368	<=	2.249634848				108.13	99.6	29.4	317				
12		4	Oak Park	1.29899696	<=	2.212232231				106.38	96	26.4	291				
13		5	York	1.3175876	=<=	1.317587603				62.4	96.2	27.2	295				
14		6	Elgin	1	=<=	1				47.19	79.9	25.5	222				
15						2) Stage informatio	n										
6	There a	re no	stage 0 variat	oles.		Describe it as a Sce	nario	Planni	ng Model								
7	We trea	t the	"Case" as a ra	ndom		WBSP RAND	Dec	lare th	e 'Case' cell to	be 'Random'	in stage 1.						
8	variable	from	1 to 6. In sta	ae 1.		WBSP VAR	Dec	lare th	e Weights to be	e Stage 1 reco	ourse variab	les.					
9	once we	see	the case we c	hoose					3								
0	wats to	try to	make the sele	ected		3) Distribution info	matic	on Ra	ndom variable	"Case" is dis	stributed fro	om 1 to 6.					
21	school	nok a	s efficient as r	nossible		WBSP DIST DISC	RETE	De	clare that 'Case	has a discret	e distributio	n over the f	nossible c	ases			
22	with the	cons	traints that no	school using t	he	(1) Sample size					o distributio		00000000	4505			
2	come w	oight	can have offi		fficio	WRSD STSC	Ma	wantt	o try all 6 scena	rios							
1	Same w	cigitta	s can have chi	cicicy > 1. L	melei	1	VVC		o try all o scene	1103.							
24						E) Deport on the or		inter	ot Efficiency	9 woighto							
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Report: DEA Efficiency

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7		York	1	0.001884	0.009173	0.0005	0.003344				
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Ideas and Steps:

Uniform Random Number Generation

Arbitrary Distribution from Uniform

Variance Reduction, Quasi-random Numbers, Super Uniforms Latin Hypercube Sampling, Antithetic Variates.

Correlated Random Numbers





LINDO API and What'sBest 10 provide:

- 1) Linear congruential, 31 bit,
- 2) Composite of linear congruentials with a long period,(default)
- 3) Mersenne Twister with long period.





IX = 742938285 * IX MOD 2147483647LSrand = IX/2147483647.0

The starting seed for the random number generator, regardless of which generator is used, can be selected by clicking on:

Add-Ins | WB! | Options | Stochastic Solver | Seed for Random Number Generator



Random Numbers from Arbitrary Distributions

Generating a random number from an arbitrary distribution, e.g., Normal, Poisson, Negative binomial...

1) Generate a uniform random number in (0, 1).

2) Convert the uniform to the desired distribution via the inverse transform of the cdf(cumulative distribution function.



Need to be able to invert u = F(x) to $x = F^{-1}(u)$.

There are lots of methods for generating r.v.'s from a given distribution. Why use the inverse transform method? **Additional Distributional Details**

Distributions supported:

(Emprical Multi-variate) DISCRETE, DISCRETE W BETA LOGARITHMIC LOGISTIC BINOMIAL CAUCHY LOGNORMAL CHISQUARE NEGATIVEBINOMIAL EXPONENTIAL NORMAT. F DISTRIBUTION PARETO GAMMA POISSON GEOMETRIC STUDENTS T GUMBEL TRIANGULAR HYPERGEOMETRIC UNIFORM LAPLACE WEIBULL

Correlations supported: Pearson, Spearman, Kendall LINDO SYSTEMS ING

Sampling: Latin Hypercube

If we need more than one observation from a univariate distribution, use Latin Hypercube sampling.

Basic idea: If taking a sample of size *N*, choose one draw randomly from each *N*th percentile. This is easy to do if Inverse Transform Method is used.

Key feature: A given possible outcome has a probability of being chosen equal to its population probability. So the sample is an unbiased sample.



Latin Hypercube Sampling

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1	А	В	C D	EF	G H	1	J	K		L	М	
1	Stochastic/Scenario Optimization of	Newsv	endor Proble	m in Wha	t'sBest (L	inear	version)					
2	Given all costs and prices, in											
3	Stage 0 we must decide how many newspapers	to stock.	In									
4	Stage 1, in the beginning, unknown demand is re	evealed to	us, and finally in									
5	Stage 1, at the end, we compute our sales and	the resulti	ing profit.									
6	1) Core model:											
7	CP = Purchase cost/unit=	30										
8	H=Holding cost/(unit leftover)=	10					Add stochastic d	ata here				
9	P=Shortage cost/(unit unsatisfied demand)=	5				2) Sta	age information					
10	V=revenue per unit sold=	60					WBSP_VAR		(Sis a stage	e 0 decision)		
11	S=Stock level(stage 1 decision)=	4.346	<<== Stage 0 de	ecision.					3) Distributi	on informati	on	
12	D=Demand(stage 2 random variable)=	8.9722	<<== Stage 1 ra	andom deman	nd.		WBSP_RAND		WBSP_DIST	LUNIFORM		
13	LS= Lost sales=	4.6262	<<== Stage 1 (r	ecourse) dec	cision.					0	Lower Li	imit
14	[LSGE] LS >= D - S (constraint)	=>=	<<== Stage 1 c	onstraint.			WBSP_VAR			10	Upper Li	imit
15	[IDEF] I=Inventory=S-D+LS=	0	<<== Stage 1 de	ecision and o	constraint.							
16	[IGE0] I >= 0 (constraint)	=>=	<<== Stage 1 no	on-negativity	constraint.	4) Sa	mple size Stage	Scenarios				
17	[TCDEF] TC = Total cost of goods = CP * S =	130.38	<<== Stage 0 co	ost computat	ion.		WBSP_STSC					
18	[THDEF] TH = Total Holding cost=H*I =	0	<<== Stage 1 h	olding cost c	omputation.			1 10				
19	[TSDEF] TS = Total Shortage cost= P*LS=	23.131	<<== Stage 1 sl	nortage cost	computation	1.						
20	[VIDEF] VI = Revenue = V*(D-LS)=	260.76	<<== Stage 1 re	evenue comp	utation.							
21	Profit, expected value, [To be maximized] =					5) Re	porting cells					
22	[TPDEF] TP = VI - TC - TH - TS =	107.249	<<== Stage 1 e	xpected valu	e (maximize))	WBSP_REP					
23												
24	Overview:											_
25	The user enters only a generic scenario 1.	aanaa" du	ving model gener	ation with th	a additional	faatura	a that					
20	a) Colle designated as stage 0 desision verich	cenes du	ning model genera	auon, with th		reature	s uidi.					
21	 a) Cells designated as stage 0 decision variable b) Cells designated as stage 1 random variable 	es are cor	strained to be et	ual in all SCE	enarios,	andom	variable in each	coonario				
20	c) the behind the scenes objective is to maximi-	ze net prot	fit averaged over	all scenario		anuom	variable in each	scenario				
30	of the berning the scenes objective is to maximiz	Le net più	ni averageu over	an scenario:								-
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LHS Illustrated, Notice "Super uniformity"

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26	- 5-	4.309918	4.30992	0	129.29755	4			
27	- 6-	4.309918	2.79945	0	23.56458	1			
28	- 7-	4.309918	9.41034	5.100424	103.79543	2			
29	- 8-	4.309918	8.37275	4.062836	108.98337	6			
30	- 9-	4.309918	1.33699	0	-78.80718	7			
31	- 10-	4.309918	6.72890	2.418982	117.20264	6			
32		-							
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Latin HyperCube vs. Simple Random Sampling

Generated a sample of 100 Normal demands with Mean = 100, SD = 10;

Histogram, Simple Random Sampling







LINDO SYSTEMS INC.

If n = sample size, there is an optimistic "optimization" bias of the order of (n-1)/n in the objective function value from <u>simple</u> SP.

LHS Benefits, Optimistic Bias of Estimates from SP

Using LHS tends to reduce this bias, as well as the variance of the estimate. Some examples:

Sir	nple random	sampling	LHS	
Problem	Mean	S. Error	Mean	S. Error
Newsvendor ⁽¹⁾				
Min cost, <i>n</i> =1000,	5546.7	28.83	5547.2	9.86
r = 100;				
Multi-product inv. ⁽¹⁾ with random yield and partial substitution	189902 n, - 100	3162	189173	1275
Max profit, $n = 256, r$	= 100			

⁽¹⁾Yang, 2004.





Three ways of measuring correlation:

Pearson

Define:

$$\overline{x} = \sum_{i=1}^{n} x_i / n; \qquad s_x = \sqrt{\sum_{i=1}^{n} (x_i - \overline{x})^2 / (n-1)};$$
$$\rho_s = \sum_{i=1}^{n} (x_i - \overline{x})(y_i - \overline{y}) / (ns_x s_y);$$

Spearman Rank

Same as Pearson, except x_i and y_i replaced by ranks, Minor adjustments when there are ties.

Kendall Tau Rank

$$\rho_{\tau} = \sum_{i=1}^{n} \sum_{k=i+1}^{n} 2* sign[(x_i - x_k)(y_i - y_k)] / [n(n-1)]$$

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Advantages of Rank, and Copulas

If two random variables are Normal distributed, then it is relatively straightforward to generate them so they have a specified correlation (Pearson).

Challenge:

If two random variables have an arbitrary distribution, it is not so easy to give them a specified correlation.

Things are easy if we use rank correlation.

The rank correlation of two random variables is unchanged by a monotonic increasing transformation, e.g., Generating

Normal random variables from Uniform random variables by the inverse cdf transformation method

does not change the rank correlation of the random variables.

The transformed Normals have the same rank correlation as the original uniforms.

The <u>Gaussian Copula</u> is a way of generating set of d random variables, each with <u>arbitrary</u> marginal distribution, but having a specified d by d rank correlation matrix.

Procedure:

- 1) Generate a sample of size *n* of *d* Normal random variables having a specified rank correlation matrix. This is relatively easy.
- 2) Convert each of the *d* Normal random variables to uniforms with the transformation: $u_{ij} = F_{normal}(x_{ij})$.
- 3) Convert each uniform to the desired target marginal distribution with the inverse transform: (Steps 2 & 3 preserve rank correlation.)

$$y_{ij} = F_j^{-1}(u_{ij}).$$

The Gaussian Copula has been named as a culprit in the mortgage securities meltdown because of false confidence in a math model.....

Kendall vs. Spearman Rank Correlation

+The Kendall correlation has a simple probabilistic interpretation.

If (x_1, y_1) and (x_2, y_2) are two observations on two random variables that have a Kendall correlation of ρ_k , then the probability that the two random variables move in the same direction is $(1 + \rho_k)/2$. That is:

Prob{ $(x_2 - x_1)^*(y_2 - y_1) > 0$ } = $(1 + \rho_k)/2$.

For example, if the weekly change in the DJI and the SP500 have a Kendall correlation of 0.8, then the probability that these two indices will change in the same direction next week is (1+0.8)/2 = 0.9.

+The Spearman coefficient seems to be finer grained.

E.g., the possible values for various sample sizes are:

Sample	Ke	ndall	Spe	arman
size	#Outcomes	Possible values	<u>#Outcomes</u>	Possible values
2	2	-1, +1	2	-1, +1
3	4	-1, -1/3, +1/3, +1	4	-1, -1/2, +1/2 1
4	7	-1, -2/3,, +2/3, 1	11	-1, -4/5,,+4/5, +1
5	11	-1, -4/5,, +4/5, +1	21	-1, -9/10,, +9/10, +1
6	16	-1, -91/105, -77/105,,+1	36	-1, -99/105, -93/105, <u>.</u> ,+1
		· 1		LINDO SYSTEMS INC.

Also, Spearman matrix is always positive definite.

Correlation Specification in What'sBest

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Correlation Specification, cont.

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EVPI (Expected Value of Perfect Information)

= Expected increase in profit if we know the future in advance.

EVMU (Expected Value of Modeling Uncertainty)

= Expected decrease in profit if we replaced each random variable by a single estimate and act as if this value is certain.

Typical single estimate is the estimated mean. Why might you rather use the median?*



*We estimate that country X will have 1.823 aircraft carriers in 2012. LINDO SHSTEMS INC.

Expected Value of Better Modeling and/or Forecasting

EVMU and EVPI are provided in What's*Best*! 10 for the Newsvendor model considered previously. The solution summary section is:

Objective (EV):	2109.684
Wait-and-see model's objective (WS):	2799.685
Perfect information (EVPI = $ EV - WS $):	690.0007
Policy based on mean outcome (EM):	2081.542
Modeling uncertainty (EVMU = $ EM - EV $):	28.14211





EVPI and EVMU: A Capacity Planning Example

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EVPI and EVMU: Capacity Planning Example Output

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EVPI Computations: Capacity Planning Example If we know future only probabilistically.. Expected total profit = 82.40 Plants to open: ATL "Wait and See" Analysis, Perfect Information: If we know scenario is 1, then Profit= 142.00 (Probability=0.3) Plants to open: STL If we know scenario is 2, then Profit= 78.00 (Probability=0.3) Plants to open: CTN If we know scenario is 3, then Profit= 57.00 (Probability=0.4)

Plants to open: CIN

Expected Profit with Perfect Information 88.80 (=.3*142 + .3*78 + .4* 57) Simple Expected Profit 82.40 Expected Value of Perfect Information(EVPI) = 6.40

Notice Atlanta not optimal for any scenario!





Plants to open:

CIN

Actual expected profit with this configuration= 71.7

Expected Profit Modeling uncertainty= 82.40 Expected Profit using expected values= 71.70 Expected Value of Modeling Uncertainty= 10.70





If EVPI = 0 does this mean the value of doing SP = 0?

....we can buy this flexible facility for just a little more...





Can we predict when EVMU = 0?

E.g., Situation 1:

The price we get for our products are random variables.

Situation 2:

The demands for our products are random variables.



The default is to use the Mean.

- + Mean is intuitive for most people.
- -Mean is undefined for some distributions, e.g., Cauchy. Median is always defined for univariate distributions.
- -Mean may not make sense for some situations,e.g., discrete distribution. The average result of roll of a die is 3.5.A fractional mean may not make sense. Median can always be chosen to be an actual possible outcome.



EVMU and EVPI, True vs. Estimated

A fine point: If the true number of scenarios is large, or infinite, and we use sampling, then the values for EVPI and EVMU reported are estimates rather than true values.



How confident should we be statistically, of the results of an SP optimization?

Issue 1) There is an optimistic bias of the order of (n-1)/n in the objective function value from an SP optimization. The optimization chooses the policy best for the sample observed.

Issue 2) If we use Latin Hypercube sampling, then the samples are correlated*, so an estimate of standard deviation among the samples based on the assumption of independence is wrong.

For modest size sample sizes, these two effects can be notable. See the next slide for example.

*Generally negatively correlated. An observation or result far below the median will be compensated by an observation far above.


		<u>A</u>	<u>p</u>	proximate Confidence Intervals, an Example
! Nev	ws	vendoi	c r	nodel;
MU =	=	1000;	1	Mean demand for the one period;
SD =	=	300;	1	Standard deviation in demand;
V =	=	140;	1	Revenue/unit sold;
C =	=	60;	1	Cost/unit purchased;
P =	=	0;	1	Penalty/unit unsatisfied demand;
H =	=	-40;	1	Holding cost/unit left in inventory;
N =	=	15;	1	Number of scenarios sampled in the SP optimization.
				The 15 is chosen for illustrative purposes only, not necessarily
				a recommended sample size;

We repeated or replicated the above 15-sample SP 1000 times. For each replication we computed a) the observed average profit, *xbar*;

b) the traditional "unbiased" estimate of the population standard deviation by

 $[\Sigma_i (x_i - xbar)^2/(n-1)]^{0.5}$, and,

c) a 90% coverage interval for *xbar*, estimating the standard deviation of *xbar* by

 $s = [\Sigma_i (x_i - xbar)^2 / (n(n-1))]^{0.5}.$

For each replication we recorded whether the computed confidence interval in fact covered the true expected profit of \$71,601. Results for the 1000 replications are shown below.

Sampling	Mean	Mean sample	Actual 90% confidence
method	<u>profit</u>	standard deviation	interval coverage
Random	\$72,127	\$25,945	.898
LHS	\$71,595	\$26,761	
True/Analytical	\$71,601		LINDU 545 IEMS INC. 💻

Some things to note:

1) Because of the modest* number of scenarios, n = 15, SP with simple random sampling seriously overestimates the expected profit by \$526. SP with LHS actually, by chance, slightly underestimates, by \$6, the true expected profit.

2) The sample standard deviation under LHS is substantially less of an underestimate of the (unknown) population standard deviation in profit than is that under simple random sampling.
3) The confidence intervals computed under simple random sampling do not quite achieve the desired 90% coverage, perhaps because the intervals are not correctly centered because of the optimistic bias in *xbar*.

4) The confidence intervals from SP with LHS are extremely conservative, and in fact achieve 100% coverage,



* Roughly, a bias of n/(n-1).



Which <u>alternative</u> investment : A, B, C, or D do you prefer?



Probabilities: A) .8, .2; B) .5, .5; C) .2, .8; D) 1.0. What are mean and s.d.?





U(w) = utility or value of having wealth w,

```
When w is a random variable, we want to
maximize E[U(w)].
Qualitatively, if
E[w_1] = E[w_2]
but w_1 is "riskier" than w_2, what would we expect about
E[U(w_1)] vs. E[U(w_2)]?
```

Reasonable features of *U*(): F1) Monotonic (strictly?) increasing. "More is better", Implies: a dominated random variable cannot be preferred.

F2) Concave(strictly?)

"Next *\$* not as useful as the previous *\$*"





May also specify a threshold *t*, and parameter *b*.

1) Downside: $U(w) = w - b * \max(0, t - w); \quad 0 \le b \le 1;$



- 2) Quadratic: $U(w) = w b^{*}(t-w)^{2}; \qquad 0 \le b;$
- 3) Power: $U(w) = (w^b 1)/b;$ $b \le 1;$

4) Log: $U(w) = \log(w)$, (Limit of Power utility as $b \rightarrow 0$); so-called "Kelly criterion". LINDO SYSTEMS INC. Plant configuration decisions, GM had too much capacity. Needed to close or refocus an unknown number of plants.

Essential Structure:

Maximize expected profit contribution – cost of reconfiguration;

GM Model: Capacity Planning Under Uncertainty

Cannot produce more in a plant than installed capacity;

Cannot sell more of a product than is demanded in a scenario.



GM SP Model, Special Features & Computations

+ Unsatisfied demand for a product transfers to other products according to a substitution matrix. One dozen products.

Key parameters:

- $c_{pv} = \text{cost per unit to produce vehicle } v \text{ in plant } p \text{ (only possible if plant is open),}$
- τ_{vw} = fraction of unsatisfied demand for vehicle v that transfers to vehicle *w*, (data from surveys),
- $CAP_{p\sigma}$ = capacity of plant *p* in configuration σ ,

Key variables:

 x_{spv} = number of units of vehicle v produced in plant p in scenario s.

Other features:

- + Infinite final period.
- + Downside risk



The key constraints in words are:

For each scenario *s* For each product (or vehicle) *v*: $Production_{vs} + Unsat_{sv} = Demand_{sv} + Transfer_in_{sv}$;

For each vehicle v and w in scenario s: $Transfer_from_to_{svw} \leq \tau_{vw} * Unsat_{sv};$

For each plant *p* and configuration σ : $Total_production_{sp} \leq CAP_{p\sigma} * y_{p\sigma}$





$penalty_s \ge threshold - profit_s$;

Expected downside risk constraint:

 $\sum_{s} Prob_{s} penalty_{s} \leq tolerance;$

Both *threshold* and *tolerance* are parameters.





Effect of putting a constraint on Downside Risk $\sum_{s} Prob_{s} penalty_{s} \leq tolerance;$





Airline Crew Scheduling, Deterministic Case

Approach used by many(most?) major airlines: Enumerate all interesting work patterns for a crew for a work period, e.g., day, week.

Variables:

 $y_p = 1$ if crew work pattern p is used.

A work pattern is a sequence of flight legs. Parameters:

 $a_{in} = 1$ if work pattern p includes flight leg i,

The deterministic, core model:

 $\operatorname{Min} \Sigma_p c_p y_p;$

For each flight segment *i*, it must be covered by some pattern *p*: $\Sigma_p a_{ip} y_p = 1$; Stage 1b constraints, for each scenario *s*:



Airline Crew Scheduling Under Uncertainty

A <u>triggering delay</u> may occur on a flight leg because of bad weather, equipment failure, etc.

A <u>cascade delay</u> can occur on a flight leg because of an earlier delay of one of the <u>three entities* needed to execute a flight leg</u>.

The SP approach (Air New Zealand, Yen & Birge)

Stage 0: Select a set of work patterns to use, the y_p .

Stage 1a: Random triggering delays occur.

Stage 1b: Compute the implied cascade delays and their costs.

*Plane, crew, passengers





How can the crew schedule chosen affect (cascade) delays?

If a flight leg is delayed(triggering or cascade), it could directly delay up to three immediately following flight legs:

- 1) A flight leg that needs the same plane,
- 2) A flight leg that needs the same crew,
- 3) A flight leg that needs a significant number of the same passengers.

If a work pattern keeps the crew on the same plane between two successive flight legs, then type 2 delay does not cause additional delay. So good work patterns from an uncertainty point of view keep the crew on the same plane.



Airline Crew Scheduling Under Uncertainty, details

Parameters:

R = set of leg pairs (i,j) for which *i* must arrive before *j* departs, because of plane or passengers,

 $w_{ijp} = 1$ if leg *i* provides the crew for leg *j* under pattern *p*,

Stage 1a random parameters:

 t_{is} = total flight time of leg *i* under scenario *s*,

Stage 1b decision variables:

 d_{is} = departure time of leg *i* under scenario *s*,

 r_{is} = arrival time or "ready for next leg" time of leg *i*, scenario *s*,

Stage 1b constraints, for each scenario *s*:

 $\begin{array}{ll} r_{is} \geq d_{is} + t_{is}, & | \text{ Flight time;} \\ d_{js} \geq r_{is} & \text{for } i,j \text{ in } R; & | \text{ Plane connection;} \\ d_{js} \geq \sum_{i} \sum_{p} w_{ijp} y_{p} r_{is}; & | \text{ Crew connection (can be linearized);} \\ \text{LINDO SYSTEMS INC.} \end{array}$

! Minimize weighted combination of explicit cost + delay, where θ specifies the tradeoff between explicit costs and delays; $\operatorname{Min} \Sigma_p c_p y_p + \theta \Sigma_i \Sigma_s d_{is};$

Airline Crew Scheduling Under Uncertainty, Full Formulation

! Stage 0 decisions and constraints,For each flight segment *i*, it must be covered by some pattern *p*:

$$\Sigma_p a_{ip} y_p = 1;$$

$$y_p = 0 \text{ or } 1;$$

! Stage 1b constraints, to compute departure times, d_{is} , as a result of random leg times, t_{is} , for each scenario s...; $r_{is} \ge d_{is} + t_{is}$, ! Ready time = departure + flight time; $d_{js} \ge r_{is}$ for *i*, *j* in *R*; ! Plane connection;

 $d_{js} \ge \sum_i \sum_p w_{ijp} y_p r_{is}$; ! Crew connection (can be linearized); LINDO SYSTEMS INC. **Stochastic Complication:**

The composition (% C, %Si, %Cr, % Mn, etc.) of input materials, typically scrap, is a random parameter, i.e., known only approximately.

Stage 0:

Choose amounts x_j of various input materials, each containing a random fraction a_{ij} of target component *i* so as to approximately get mixture into target interval for component *i*.

Stage 1, beginning:

Melt mixture and observe actual composition for each *i*;

Stage 1, end:

Add additional, more pure and more expensive materials to move any wayward quality measures to within tolerance.

Recourse decision must be quick, < 1 min.





This presentation benefited from the comments of *Sue Lisowski*.





Atlihan, M., K. Cunningham. G. Laude, and L. Schrage(2010), "Challenges in Adding a Stochastic Programming/Scenario Planning Capability to a General Purpose Optimization Modeling System", in *A Long View of Research and Practice in Operations Research and Management Science*, Springer, vol. 148, editors Sodhi, M. and C. Tang, pp.117-134.

