Vehicle Routing Optimization

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Keywords: Vehicle Routing, Routing, LTL Delivery, VRP,

An important practical problem is the routing of vehicles from a central depot, -the so-called Vehicle Routing Problem (VRP). Examples abound: the routing of delivery trucks for a parcel delivery service, routing of appliance deliveries, routing of repairmen, etc. The VRP is like a traveling salesman problem but with multiple salesmen with capacity constraints for each salesman and associated trip. There are typically two constraints on a trip: a) vehicle load, e.g., a vehicle can carry at most 24 pallets, and b) trip length constraints, e.g., at most eight hours. This problem is sometimes also called the LTL(Less than Truck Load) routing problem because a typical recipient receives less than a truck load of goods.

It is worth noting that the VRP is very similar to a common task scheduling problem:
 Given a set of tasks to be performed, and a
  set of machines on which the tasks can be performed, and perhaps a
  time limit, e.g., 8 hours, on how much time is available on each machine, and perhaps,
  changeover time matrix between tasks,
Which tasks should be assigned to which machines, and in which order should the tasks be done on each machine?

Example:
We need to deliver goods to 17 different cities from a depot in Dallas. We have a fleet of trucks for making the deliveries. Each truck can carry up to 24 pallets. The receiving cities, along with the number of pallets needed, and the latitude and longitude of each city are listed below.

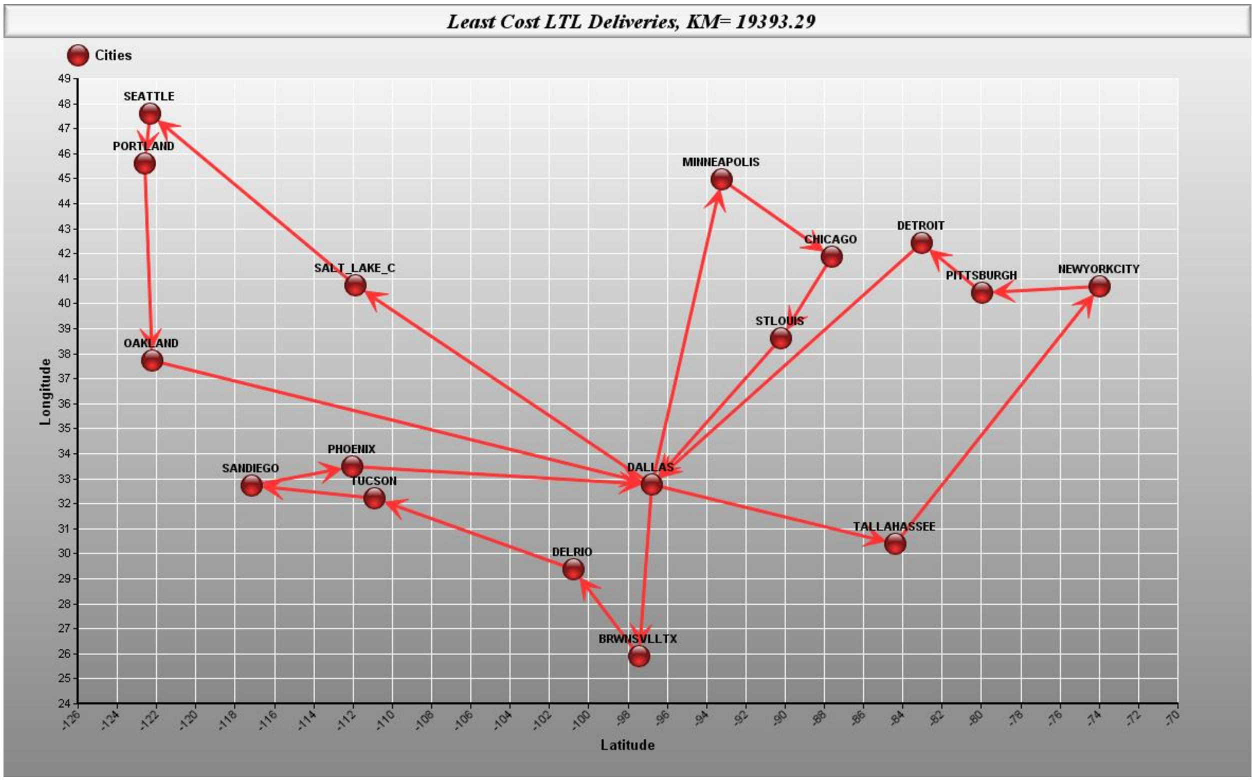
```
  CITY          Q     LATI       LNGT   =
BrwnsvllTX      4   25.9000    -97.4333
Chicago         7   41.8781    -87.6298
DelRio          2   29.3667   -100.7833
Detroit         3   42.4167    -83.0167
Minneapolis     6   44.9800    -93.2519
NewYorkCity     9   40.7128    -74.0059
Oakland         9   37.7297   -122.2200
Phoenix         5   33.4833   -112.0667
Pittsburgh      5   40.4406    -79.9959
Portland        4   45.5997   -122.5997
Salt_Lake_C     8   40.7500   -111.8833
SanDiego        4   32.7333   -117.1667
Seattle         2   47.6062   -122.3321
StLouis        11   38.6270    -90.1994
Tucson          6   32.2217   -110.9258
Tallahassee     5   30.3833    -84.3667
Dallas          0   32.7767    -96.7970   ;

Depot = Dallas; ! Identify the depot city;
Vcap = 24; ! Vehicle capacity in pallets allowed on vehicle;
```

Even though Chicago, Detroit, Minneapolis, and St. Louis are moderately close, they cannot be on the same trip because their total demand of 7+3+6+11 = 27 > 24. The typical optimization problem associated with the VRP is to:

Minimize the total distance traveled over all trips
Subject to:
Each city receives its demanded quantity from some trip, and
Load on each trip does not exceed the vehicle capacity.

The total distance travelled is minimized by the using the four trips shown graphically below.



More details on the four trips are shown in the report below:

ROUTE 1:

FROM	TO	DISTANCE	LOAD
DALLAS	BRWNSVLLTX	767.1	4.0
BRWNSVLLTX	DELRIO	1274.5	6.0
DELRIO	TUCSON	2293.5	12.0
TUCSON	SANDIEGO	2881.6	15.0
SANDIEGO	PHOENIX	3363.8	20.0
PHOENIX	DALLAS	4786.6	20.0

```
ROUTE 2:
   FROM        TO            DISTANCE      LOAD
------------------------------------
   DALLAS      MINNEAPOLIS     1390.8       6.0
   MINNEAPOLIS CHICAGO         1960.8      13.0
   CHICAGO     STLOUIS         2382.9      24.0
   STLOUIS     DALLAS          3264.5      24.0

ROUTE 3:
   FROM        TO            DISTANCE      LOAD
------------------------------------
   DALLAS      SALT_LAKE_C     1606.4       8.0
   SALT_LAKE_C SEATTLE         2734.0      10.0
   SEATTLE     PORTLAND        2958.0      14.0
   PORTLAND    OAKLAND         3833.7      23.0
   OAKLAND     DALLAS          6199.0      23.0

ROUTE 4:
   FROM        TO            DISTANCE      LOAD
------------------------------------
   DALLAS      TALLAHASSEE     1206.4       5.0
   TALLAHASSEE NEWYORKCITY     2686.6      14.0
   NEWYORKCITY PITTSBURGH      3193.3      19.0
   PITTSBURGH  DETROIT         3527.5      21.0
   DETROIT     DALLAS          5143.3      21.0
```

The total distance travelled by this solution is 19393.29 km. The longest trip is 6199 km.

You can solve small problems such as the above with the model, VRouteMTZplusTM.lng, available in the MODELS library at www.lindo.com. You can ask interesting questions, such as, suppose you restrict the maximum length of any trip to 6000 km. How much will the total distance increase?

An obvious question is: How much can be saved by switching to using optimal routes, relative what we are doing currently? One estimate is to compare the cost of an optimal route to a good heuristic route. A standard, good heuristic is the Clarke-Wright Savings heuristic (CWSH). It is probably typical of the kind of solution one would get if you solved the VRP manually. For the above example, the CWSH gives a solution with a total distance of 20368.42 km. This is a percentage savings of
(20368.42 – 19393.29)/19393.29 = 0.05, i.e., a savings of 5%.

The crucial set of data in a VRP is the distance matrix. For the example above we computed distances based on the latitude and longitudes of the cities, using Great Circle distances. Many times, it is not distance that is important but rather travel time. Travel times take rather more time to estimate, but they may give a more accurate representation of the real problem. Summarizing, the typical ways of supplying the distances are: i) Explicitly. The default is that the distance matrix is in "from-to" form, but sometimes the distances are in transposed "to-from" form. If the matrix is symmetric, i.e., distance from i to j = distance from j to i, then it does not matter. ii) via x-y coordinates of each location, The default is to calculate distances as Euclidean (as the crow flies), or as "Manhattan" or L1 distances, as appropriate if all the streets are on a grid, as in Manhattan and there are no diagonal streets. iii) via latitude-longitude coordinates of each location. In this case "Great circle" distances are calculated. For short distances near the equator, these distance are close to Euclidian distances. For longer distances, say as flown by an airplane, or farther away from the equator, Great Circle distances are more accurate. It may be convenient to have a semi-permanent big distance matrix. The routing problem encountered each day may involve only a subset of the cities, so one wants to have a way of specifying which subset of the cities are involved in today's problem.

Some other variations on the basic problem are listed below.
a) **Multiple dimensions to capacity**. In addition to a vehicle capacity constraint, e.g., at most 24 pallets, there is frequently a trip length constraint, e.g., eight hours. There may be a weight constraint, as well as a volume, "cube" constraint.
b) **Nonzero Visit Time**: If you are routing appliance delivery or repairmen, and there is a constraint on total trip time, then you want to take into account the time taken at each stop.
c) **Time** windows. If there are limits on when each customer can be visited, then the problem may be a lot more difficult;
d) **Multiple vehicle types**: An "18-wheeler" may be able to make deliveries to a suburban address but cannot maneuver in a downtown area, so a delivery service may need multiple delivery vehicle types.
e) **Dynamic distance matrix**: For some regions, e.g., where there is a heavy morning or evening commute, the travel time depends significantly upon the time of day. This can be handled in part with time windows to avoid deliveries to certain regions during heavy traffic hours.
f) **Split deliveries**: If the demand at a customer is greater than vehicle capacity, then a split delivery is unavoidable. Even if every city demand is smaller than vehicle capacity, nevertheless allowing split deliveries may reduce the total distance. Suppose you have 3 cities near each other, but distant from the depot, each with demand 16, and vehicle capacity = 24. If split deliveries are not allowed you will have to send 3 vehicles. If split deliveries are allowed, you need only two.
g) **Non-Symmetric distance matrix**: If you are using a public carrier, it may be that you have to pay only for the distance to the final stop. You do not have to pay for the final trip leg back to the depot from the final stop. The distance back to the depot is effectively zero. If you are sequencing tasks on various machines, the changeover time matrix is typically not symmetric.
h) **Multiple depots**. It is typically 1, but if you are scheduling pickups, there may additionally be a choice of which depot serves which customer. This feature may make the problem harder.
i) **Uncertainty:** If you are scheduling home repair or home delivery personnel, you frequently do not know with certainty the time at each stop. Similarly, travel times may be affected by weather or unexpected congestion. One approach is to estimate the variance of each step, sum the variances in a trip and constrain it. Also, in practice, high variance stops or legs are placed towards the end of the trip.
j) **Load building**: There is an interaction between vehicle routing and the loading of the vehicles. Trucks are typically unloaded from the back. Thus, when loading the truck for deliveries, you may want to load it in LIFO (Last In First Out) order. Complicating matters, if items are heavy, you may want to load items so the weight is evenly distributed. If there are pickups and deliveries, you may want to put deliveries early in the trip.

The solvability of practical VRP's depends upon a variety of typical features. Problems with up to 20 cities can usually be solved in seconds to proven optimality with moderately simple models. Excellent solutions can typically be found quickly for problems with several hundred cities. Difficulty depends upon various features, e.g., the geometry of the region. VRP's in long narrow regions such as Chile are much easier to solve than VRP's in a region with a more uniform density of locations such as the U.S. Situations in which there are just two or three stops per vehicle tend to be easier to solve.

For more information on solving more difficult VRP's, contact www.lindo.com.